

thm_2Efinite_map_2EFUPDATE_LIST_APPLY_NOT_MEM_m
(TMSHYSE7FVv8DnFPdUnx9wiFsNba62GTxnk)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (2)$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EFUPDATE A_27a A_27b \in (((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)})^{(ty_2Efinite_map_2EFUPDATE A_27a A_27b)}) \quad (3)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (4)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDL A_27a A_27b \in (((A_27b^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{((A_27b^{A_27a})^{A_27b})}) \quad (5)$$

Definition 5 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Elist_2EFUPDATE_LIST) (c_2Elist_2EFUPDATE_LIST))$.
Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (6)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (7)$$

Let $c_2Efinite_map_2E fmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2E fmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2E fmap_REP\ A_27a\ A_27b)}) \quad (8)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a)^{(ty_2Esum_2Esum\ A_27a\ A_27b)} \quad (9)$$

Definition 6 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E3D) (2^{A_27a})) (c_2Ebool_2E21) P))$.

Definition 7 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFAPPLY)$.

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (10)$$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b)^{A_27a}}) \quad (11)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (12)$$

Definition 8 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$.

Definition 9 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E21) 2) (\lambda V0t \in 2.V0t)$.

Definition 10 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E21) 2) (\lambda V2t \in 2. (ap (c_2Ebool_2E21) 2) V2t))))$.

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\neg(\forall V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow & (\\ \forall V0kvl \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). & \\ (\forall V1f \in (ty_2Efinite_map_2Efmmap\ A_27a\ A_27b). (\forall V2k \in & \\ A_27a. ((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2k)\ (ap\ (c_2Elist_2ELIST_TO_SET & \\ A_27a)\ (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_27a\ A_27b) & \\ A_27a)\ (c_2Epair_2EFST\ A_27a\ A_27b))\ V0kvl)))))) \Rightarrow ((ap\ (ap\ (c_2Efinite_map_2EFAPPLY & \\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b) & \\ V1f)\ V0kvl))\ V2k) = (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b) & \\ V1f)\ V2k)))))) \quad (29) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (39)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0kvl \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A.27a\ A.27b)). \\ & (\forall V1f \in (ty_2Efinite_map_2Efmap\ A.27a\ A.27b). (\forall V2k \in \\ & A.27a. (\forall V3v \in A.27b. (((\neg(p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a) \\ V2k)\ (ap\ (c.2Elist_2ELIST_TO_SET\ A.27a)\ (ap\ (ap\ (c.2Elist_2EMAP \\ (ty_2Epair_2Eprod\ A.27a\ A.27b)\ A.27a)\ (c.2Epair_2EFST\ A.27a\ A.27b)) \\ V0kvl)))))) \wedge (V3v = (ap\ (ap\ (c.2Efinite_map_2EFAPPLY\ A.27a\ A.27b) \\ V1f)\ V2k))) \Rightarrow ((ap\ (ap\ (c.2Efinite_map_2EFAPPLY\ A.27a\ A.27b)\ (\\ ap\ (ap\ (c.2Efinite_map_2EFUPDATE_LIST\ A.27a\ A.27b)\ V1f)\ V0kvl)) \\ V2k) = V3v)))))) \end{aligned}$$