

thm_2Efinite_map_2EFUPDATE_LIST_SAME_UPDATE (TMZrj89GzrPddgCrrwx8jAVqRJ3CRZz2RUq)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda P \in 2^A.(ap V0P (ap (c_2Emin_2E_40 A P)))$

Definition 4 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 5 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda P \in 2^A.(ap (ap (c_2Emin_2E_3D (2^A-27a)) P))$

Definition 7 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^A-27b)^{A-27a})^2}) \tag{3}$$

Definition 11 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \quad (4)$$

Let $c_2Efinite_map_2Efmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_ABS A_27a A_27b \in ((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a}}) \quad (5)$$

Definition 12 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Efinite_map_2E$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_REP A_27a A_27b \in (((ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap A_27a A_27b)}) \quad (6)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EOUTL A_27a A_27b \in (A_27a^{(ty_2Esum_2Esum A_27a A_27b)}) \quad (7)$$

Definition 13 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EISL A_27a A_27b \in (2^{(ty_2Esum_2Esum A_27a A_27b)}) \quad (8)$$

Definition 14 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Let $c_2Efinite_map_2EDRESTRICT : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EDRESTRICT A_27a A_27b \in (((ty_2Efinite_map_2Efmap A_27a A_27b)^{(2^{A_27a}}))^{(ty_2Efinite_map_2Efmap A_27a A_27b)}) \quad (9)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (10)$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b \in (((ty_2Efinite_map_2Efm\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)})$$
 (11)

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0)$$
 (12)

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL\ A_27a\ A_27b \in (((A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b)^{A_27a}})^{A_27b}$$
 (13)

Definition 16 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap\ (c_2Elist_2Elist\ A_27a)\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b))$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b)^{A_27a}})$$
 (14)

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)})$$
 (15)

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a})$$
 (16)

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a)$$
 (17)

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}$$
 (18)

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b}})^{A_27a})$$
 (19)

Definition 17 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Elist_2Elist\ A_27a)\ (c_2Epair_2EABS_prod\ A_27a\ A_27b))\ V0x\ V1y$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (28)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t)))))) \quad (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))) \quad (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow \\
& True)) \quad (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\
& A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t)))))) \quad (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t1 \in A.27a. (\forall V1t2 \in \\
& A.27a. (((ap (ap (ap (c_2Ebool_2ECOND A.27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A.27a) c_2Ebool_2EF) \\
& V0t1) V1t2) = V1t2)))))) \quad (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). ((\neg(\forall V1x \in \\
& A.27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a. (\neg(p (ap V0P V2x)))))) \quad (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ (2^{A_27a}). ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow \\ ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A_27a. (p\ (\\ ap\ V1Q\ V4x)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((p\ V0P) \vee (\exists V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow (\exists V3x \in \\ A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ 2. ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q)))) \Leftrightarrow ((\exists V3x \in \\ A_27a. (p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((\exists V2x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow ((p \\ V0P) \wedge (\exists V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0b \in 2. (\forall V1f \in (A_27b^{A_27a}). (\forall V2g \in (A_27b^{A_27a}). \\ (\forall V3x \in A_27a. ((ap\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (A_27b^{A_27a})) \\ V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V0b)\ (ap \\ V1f\ V3x))\ (ap\ V2g\ V3x)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\ & \quad (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & \quad V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f \\ & \quad V2x))\ (ap\ V0f\ V3y)))))) \\ & \end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned} & (\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p\ (ap\ (ap \\ & \quad (ap\ (c_2Ebool_2ECOND\ 2)\ V0b)\ V1t1)\ V2t2)) \Leftrightarrow (((\neg(p\ V0b)) \vee (p\ V1t1)) \wedge \\ & \quad ((p\ V0b) \vee (p\ V2t2)))))) \\ & \end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & \quad 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\ & \quad (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \\ & \end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & \quad (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & \quad (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & \quad ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & \quad V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ & \quad V5y_27)))))) \\ & \end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & \quad A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & \quad V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & \quad (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \\ & \end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b). (\forall V1a \in \\ & \quad A_27a. (\forall V2b \in A_27b. ((ap\ (c_2Efinite_map_2EFDOM\ A_27a \\ & \quad A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V0f)\ (ap \\ & \quad (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1a)\ V2b))) = (ap\ (ap\ (c_2Epred_set_2EINSERT \\ & \quad A_27a)\ V1a)\ (ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27b)\ V0f)))))) \\ & \end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V1a \in \\
& \quad A.27a.(\forall V2b \in A.27b.(\forall V3x \in A.27a.((ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\
& \quad A.27a\ A.27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A.27a\ A.27b)\ V0f) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27b)\ V1a)\ V2b)))\ V3x) = (ap\ (ap\ (ap \\
& \quad (c_2Ebool_2ECOND\ A.27b)\ (ap\ (ap\ (c_2Emin_2E_3D\ A.27a)\ V3x)\ V1a)) \\
& \quad V2b)\ (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A.27a\ A.27b)\ V0f)\ V3x)))))) \\
& \hspace{15em} (52)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V1g \in \\
& \quad (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(((ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ V0f) = (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V1g))) \wedge \\
& \quad (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ (ap\ (\\
& \quad c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V0f)))) \Rightarrow ((ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\
& \quad A.27a\ A.27b)\ V0f)\ V2x) = (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A.27a \\
& \quad A.27b)\ V1g)\ V2x)))))) \Leftrightarrow (V0f = V1g))) \\
& \hspace{15em} (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V1r \in \\
& \quad (2^{A.27a}).(((ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ (ap\ (ap \\
& \quad (c_2Efinite_map_2EDRESTRICT\ A.27a\ A.27b)\ V0f)\ V1r)) = (ap\ (ap \\
& \quad (c_2Epred_set_2EINTER\ A.27a)\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a \\
& \quad A.27b)\ V0f))\ V1r)) \wedge (\forall V2x \in A.27a.((ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\
& \quad A.27a\ A.27b)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A.27a\ A.27b) \\
& \quad V0f)\ V1r))\ V2x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A.27b)\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27a)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A.27a)\ (ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ V0f))\ V1r)))\ (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A.27a \\
& \quad A.27b)\ V0f)\ V2x))\ (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A.27a\ A.27b) \\
& \quad (c_2Efinite_map_2EFEMPTY\ A.27a\ A.27b))\ V2x)))))) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27b).(\forall V1r \in \\
& (2^{A_27a}).(\forall V2x \in A_27a.(\forall V3y \in A_27b.((ap\ (ap\ (c_2Efinite_map_2EDRESTRICT \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V0f) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y)))\ V1r) = (ap\ (ap\ (ap \\
& \quad (c_2Ebool_2ECOND\ (ty_2Efinite_map_2E fmap\ A_27a\ A_27b))\ (ap \\
& (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1r))\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\
& \quad V0f)\ V1r)))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y)))\ (ap\ (\\
& \quad ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b)\ V0f)\ V1r)))))) \\
& \hspace{15em} (55)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27b).(((ap\ (ap \\
& (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b)\ V0f)\ (c_2Elist_2ENIL \\
& (ty_2Epair_2Eprod\ A_27a\ A_27b)))) = V0f) \wedge (\forall V1h \in (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b).(\forall V2t \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)).((ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a \\
& \quad A_27b)\ V0f)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ A_27a\ A_27b) \\
& \quad V1h)\ V2t))) = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b) \\
& \quad (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V0f)\ V1h))\ V2t)))))) \\
& \hspace{15em} (56)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& \quad V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\
& \quad (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\
& \quad A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\
& \quad (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t)))))) \\
& \hspace{15em} (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0h \in A_27b.(\forall V1t \in (ty_2Elist_2Elist\ A_27b).((\\
& (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) = \\
& (c_2Epred_set_2EEMPTY\ A_27a)) \wedge ((ap\ (c_2Elist_2ELIST_TO_SET \\
& \quad A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Epred_set_2EINSERT \\
& \quad A_27b)\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27b)\ V1t)))))) \\
& \hspace{15em} (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Elist_2Elist\ A.27a)}), \\ & (((p\ (ap\ V0P\ (c.2Elist_2ENIL\ A.27a))) \wedge (\forall V1t \in (ty_2Elist_2Elist \\ & A.27a).(p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A.27a.(p\ (ap\ V0P\ (ap\ (ap\ (\\ & c.2Elist_2ECONS\ A.27a\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty_2Elist_2Elist \\ & A.27a).(p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0x \in A.27a.(\forall V1y \in A.27b.((ap\ (c.2Epair_2EFST\ A.27a \\ & A.27b)\ (ap\ (ap\ (c.2Epair_2E_2C\ A.27a\ A.27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0P \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}).((\forall V1p \in \\ & (ty_2Epair_2Eprod\ A.27a\ A.27b).(p\ (ap\ V0P\ V1p))) \Leftrightarrow (\forall V2p_1 \in \\ & A.27a.(\forall V3p_2 \in A.27b.(p\ (ap\ V0P\ (ap\ (ap\ (c.2Epair_2E_2C \\ & A.27a\ A.27b)\ V2p_1)\ V3p_2)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool_2EIN \\ & A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\neg(p\ (ap\ (ap \\ & (c.2Ebool_2EIN\ A.27a)\ V0x)\ (c.2Epred_set_2EEMPTY\ A.27a)))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool_2EIN\ A.27a) \\ & V2x)\ (ap\ (ap\ (c.2Epred_set_2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\ & (ap\ (c.2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c.2Ebool_2EIN \\ & A.27a)\ V2x)\ V1t)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0s \in (2^{A.27a}).((ap\ (\\ & ap\ (c.2Epred_set_2EINTER\ A.27a)\ (c.2Epred_set_2EUNIV\ A.27a)) \\ & V0s) = V0s)) \wedge (\forall V1s \in (2^{A.27a}).((ap\ (ap\ (c.2Epred_set_2EINTER \\ & A.27a)\ V1s)\ (c.2Epred_set_2EUNIV\ A.27a)) = V1s))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s)))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s)))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ & (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2ECOMPL \\ & A_27a)\ V1s)))) \Leftrightarrow (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V1s)))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Epred_set_2ECOMPL\ A_27a) \\ & (c_2Epred_set_2EEMPTY\ A_27a)) = (c_2Epred_set_2EUNIV\ A_27a)) \end{aligned} \quad (68)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (70)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (71)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (72)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (73)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(\\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge ((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (74)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (75)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q) \vee (\neg(p V0p))))))
\end{aligned} \tag{78}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{81}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{82}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{83}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0kvl \in (ty_2Elist_2Elist (ty_2Epair_2Eprod A_27a A_27b)). \\
& (\forall V1f1 \in (ty_2Efinite_map_2Efmap A_27a A_27b). (\forall V2f2 \in \\
& (ty_2Efinite_map_2Efmap A_27a A_27b). (((ap (ap (c_2Efinite_map_2EFUPDATE_LIST \\
& A_27a A_27b) V1f1) V0kvl) = (ap (ap (c_2Efinite_map_2EFUPDATE_LIST \\
& A_27a A_27b) V2f2) V0kvl)) \Leftrightarrow ((ap (ap (c_2Efinite_map_2EDRESTRICT \\
& A_27a A_27b) V1f1) (ap (c_2Epred_set_2Ecompl A_27a) (ap (c_2Elist_2ELIST_TO_SET \\
& A_27a) (ap (ap (c_2Elist_2EMAP (ty_2Epair_2Eprod A_27a A_27b) \\
& A_27a) (c_2Epair_2EFST A_27a A_27b)) V0kvl)))) = (ap (ap (c_2Efinite_map_2EDRESTRICT \\
& A_27a A_27b) V2f2) (ap (c_2Epred_set_2Ecompl A_27a) (ap (c_2Elist_2ELIST_TO_SET \\
& A_27a) (ap (ap (c_2Elist_2EMAP (ty_2Epair_2Eprod A_27a A_27b) \\
& A_27a) (c_2Epair_2EFST A_27a A_27b)) V0kvl))))))
\end{aligned}$$