

thm_2Efinite__map_2EFUPDATE__LIST__THM (TMXioTP3vFTtynLsnPj9kfdha1kPmt5Q4fz)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Efinite_map_2E fmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2E fmap A0 A1) \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (2)$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EFUPDATE A_27a A_27b \in (((ty_2Efinite_map_2E fmap A_27a A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)})^{(ty_2Efinite_map_2EFUPDATE A_27a A_27b)}) \quad (3)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (4)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDL A_27a A_27b \in (((A_27b)^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \quad (5)$$

Definition 3 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap (c_2Elist_2EFOLDL$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (6)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (7)$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a})))$

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ V2t))))$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad (\forall V0f \in ((A_27b^{A_27a})^{A_27b}). (\forall V1e \in A_27b. ((ap\ (\\ & \quad ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V0f)\ V1e)\ (c_2Elist_2ENIL\ A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27a})^{A_27b}). (\forall V3e \in \\ & \quad A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\ & \quad ((ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V4x)\ V5l)) = (ap\ (ap\ (ap\ (c_2Elist_2EFOLDL\ A_27a\ A_27b)\ V2f)\ (ap\ (ap\ V2f\ V3e)\ V4x))\ V5l)))))) \end{aligned} \quad (10)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (ty_2Efinite_map_2EFmap\ A_27a\ A_27b). (((ap\ (ap\ (\\ & \quad c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b)\ V0f)\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod\ A_27a\ A_27b))) = V0f) \wedge (\forall V1h \in (ty_2Epair_2Eprod\ A_27a\ A_27b). (\forall V2t \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). ((ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b)\ V0f)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ A_27a\ A_27b)\ V1h)\ V2t)) = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b)\ V0f)\ V1h))\ V2t)))))) \end{aligned}$$