

thm_2Efinite__map_2EMAP__KEYS__BIJ__LINV (TMYUTch58K1X4Y4o1FqEnuRHMCpCVJ8BstZ)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (2)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (3)$$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2Efmap\ A0\ A1) \quad (4)$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)) \quad (5)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (6)$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2)$

Let $c_2Efinite_map_2EMAP_KEYS : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow c_2Efinite_map_2EMAP_KEYS\ A_27a\ A_27b\ A_27c \in \\ & ((ty_2Efinite_map_2Efmap\ A_27b\ A_27c)^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27c)})^{(A_27b^A_27a)} \end{aligned} \quad (7)$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL \\ & A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (8)$$

Definition 5 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2)$

Definition 6 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 7 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 8 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 9 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A_27a))\ A_27a)$

Definition 10 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1y \in A_27c$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).ap\ V1f\ V0x))$

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ & A0\ A1) \end{aligned} \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (10)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ (V0x\ V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (11)$$

Definition 15 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p \text{ (ap } P \ x)) \text{ then (the } (\lambda x.x \in A \wedge P \ x) \text{ of type } \iota \Rightarrow \iota.$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(\text{ap } V0P \ (\text{ap } (c_2Emin_2E_40$

Definition 18 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 19 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 20 We define c_2Eone_2Eone to be $(\text{ap } (c_2Emin_2E_40 \ \text{ty_2Eone_2Eone}) \ (\lambda V0x \in \text{ty_2Eone_2Eone}$

Definition 21 We define c_2Ebool_2EF to be $(\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2.V0t)).$

Definition 22 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(\text{ap } (\text{ap } c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2E_7E$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a \ A_27b \in ((\text{ty_2Esum_2Esum } A_27a \ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (12)$$

Definition 23 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(\text{ap } (c_2Esum_2EABS$

Let $\text{ty_2Eoption_2Eoption} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.\text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eoption_2Eoption } A0) \quad (13)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Eoption_2Eoption_ABS \ A_27a \in ((\text{ty_2Eoption_2Eoption } A_27a)^{(\text{ty_2Esum_2Esum } A_27a \ \text{ty_2Eone_2Eone})}) \quad (14)$$

Definition 24 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(\text{ap } (c_2Eoption_2Eoption_ABS \ A_27a) \ (c$

Definition 25 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(\text{ap } (c_2Esum_2EABS$

Definition 26 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(\text{ap } (c_2Eoption_2Eoption_ABS$

Definition 27 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V$

Definition 28 We define $c_2Epred_set_2ELINV_OPT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V$

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Eoption_2ETHE \ A_27a \in (A_27a^{(\text{ty_2Eoption_2Eoption } A_27a)}) \quad (15)$$

Definition 29 We define $c_2Epred_set_2ELINV$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 30 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^A$

Definition 31 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (27)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (28)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0f \in (2^{A_{27a}}).(\forall V1v \in A_{27a}.((\forall V2x \in A_{27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (29)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{A_{27a}}).(\forall V1Q \in 2.(((\exists V2x \in A_{27a}.(p (ap V0P V2x))) \Rightarrow (p V1Q)) \Leftrightarrow (\forall V3x \in A_{27a}.((p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \wedge (((\exists V4x \in A_{27a}.(p (ap V0P V4x))) \wedge (p V1Q)) \Leftrightarrow (\exists V5x \in A_{27a}.((p (ap V0P V5x)) \wedge (p V1Q)))))) \wedge (((p V1Q) \wedge (\exists V6x \in A_{27a}.(p (ap V0P V6x)))) \Leftrightarrow (\exists V7x \in A_{27a}.((p V1Q) \wedge (p (ap V0P V7x))))))))) \quad (30)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow \forall A_{27c}.nonempty A_{27c} \Rightarrow (\forall V0f \in (A_{27b}^{A_{27a}}).(\forall V1g \in (A_{27a}^{A_{27c}}).(\forall V2x \in A_{27c}.((ap (ap (ap (c.2Ecombin_2Eo A_{27c} A_{27b} A_{27a}) V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \quad (31)$$

Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow (\forall V0x \in A_{27a}.((ap (c.2Ecombin_2EI A_{27a}) V0x) = V0x)) \quad (32)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V1g \in \\
& (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).((V0f = V1g) \Leftrightarrow (((ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ V0f) = (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V1g)) \wedge \\
& \quad (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ (ap\ (\\
& \quad c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V0f)))) \Rightarrow ((ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\
& \quad A.27a\ A.27b)\ V0f)\ V2x) = (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A.27a \\
& \quad A.27b)\ V1g)\ V2x))))))))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1fm \in (ty_2Efinite_map_2E fmap \\
& \quad A.27a\ A.27c).(((ap\ (c_2Efinite_map_2EFDOM\ A.27b\ A.27c)\ (ap\ (\\
& \quad ap\ (c_2Efinite_map_2EMAP_KEYS\ A.27a\ A.27b\ A.27c)\ V0f)\ V1fm)) = \\
& (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A.27a\ A.27b)\ V0f)\ (ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27c)\ V1fm))) \wedge ((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EINJ\ A.27a \\
& \quad A.27b)\ V0f)\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27c)\ V1fm))\ (c_2Epred_set_2EUNIV \\
& \quad A.27b))) \Rightarrow (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a) \\
& \quad V2x)\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27c)\ V1fm)))) \Rightarrow ((ap\ (ap \\
& \quad (c_2Efinite_map_2EFAPPLY\ A.27b\ A.27c)\ (ap\ (ap\ (c_2Efinite_map_2EMAP_KEYS \\
& \quad A.27a\ A.27b\ A.27c)\ V0f)\ V1fm))\ (ap\ V0f\ V2x)) = (ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\
& \quad A.27a\ A.27c)\ V1fm)\ V2x))))))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(p\ (ap\ (ap\ (c_2Ebool_2EIN \\
\quad A.27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A.27a)))) \hspace{10em} (35)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0y \in A.27b.(\forall V1s \in (2^{A.27a}).(\forall V2f \in (A.27b^{A.27a}). \\
& ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad A.27a\ A.27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A.27a.((V0y = (ap\ V2f\ V3x)) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V3x)\ V1s))))))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).((ap\ (ap \\
\quad (c_2Epred_set_2EIMAGE\ A.27a\ A.27a)\ (c_2Ecombin_2EI\ A.27a)) \\
\quad V0s) = V0s)) \hspace{10em} (37)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27c^{A_27b}). (\forall V1g \in (A_27b^{A_27a}). \\
& (\forall V2s \in (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a \\
& A_27c)\ (ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27c\ A_27b)\ V0f)\ V1g))\ V2s) = \\
& (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& A_27a\ A_27b)\ V1g)\ V2s))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in \\
& (2^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EINJ\ A_27a\ A_27b)\ V0f) \\
& V1s)\ V2t)) \Rightarrow (\forall V3x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& V3x)\ V1s)) \Rightarrow ((ap\ (ap\ (c_2Epred_set_2ELINV\ A_27a\ A_27b)\ V0f) \\
& V1s)\ (ap\ V0f\ V3x)) = V3x))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in \\
& (2^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ\ A_27a\ A_27b)\ V0f) \\
& V1s)\ V2t)) \Rightarrow (\forall V3x \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b) \\
& V3x)\ V2t)) \Rightarrow ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Epred_set_2ELINV\ A_27a\ A_27b) \\
& V0f)\ V1s)\ V3x)) = V3x))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in \\
& (2^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ\ A_27a\ A_27b)\ V0f) \\
& V1s)\ V2t)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ\ A_27b\ A_27a)\ (ap \\
& (ap\ (c_2Epred_set_2ELINV\ A_27a\ A_27b)\ V0f)\ V1s))\ V2t)\ V1s))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{42}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (53)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). \\ & (\forall V1t \in (ty_2Efinite_map_2E fmap ty_2Enum_2Enum A_27a). \\ & ((p (ap (ap (ap (ap (c_2Epred_set_2EBIJ ty_2Enum_2Enum ty_2Enum_2Enum) \\ & V0f) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)) (c_2Epred_set_2EUNIV \\ & ty_2Enum_2Enum))) \Rightarrow ((ap (ap (c_2Efinite_map_2EMAP_KEYS ty_2Enum_2Enum \\ & ty_2Enum_2Enum A_27a) V0f) (ap (ap (c_2Efinite_map_2EMAP_KEYS \\ & ty_2Enum_2Enum ty_2Enum_2Enum A_27a) (ap (ap (c_2Epred_set_2ELINV \\ & ty_2Enum_2Enum ty_2Enum_2Enum) V0f) (c_2Epred_set_2EUNIV ty_2Enum_2Enum))) \\ & V1t)) = V1t)))))) \end{aligned}$$