

thm_2Efinite_map_2ESTRONG_DRESTRICT_FUPDATE_THM
(TM-
caSWgaWi9kTZo2yxjMPSSGSWpoeCFg2a5)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$
of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 6 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \tag{3}$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b \in (((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)})$$
(4)

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone$$
(5)

Definition 7 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 8 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$.

Definition 9 We define c_2Ebool_2E21 to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21))$.

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1)$$
(6)

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2})$$
(7)

Definition 11 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$.

Let $c_2Efinite_map_2Efmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_ABS\ A_27a\ A_27b \in ((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a}})$$
(8)

Definition 12 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Efinite_map_2Efmap_ABS\ A_27a\ A_27b)\ 0)$.

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap\ (c_2Efinite_map_2EFEMPTY\ A_27a\ A_27b)\ V2t2)\ V1t1)))$.

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)})$$
(9)

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)})$$
(10)

Definition 14 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFmap A_27a A_27b)$

Let $c_2Efinite_map_2EDRESTRICT : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EDRESTRICT \\ & A_27a A_27b \in ((ty_2Efinite_map_2Emap A_27a A_27b)^{(2^{A-27a})})^{(ty_2Efinite_map_2Emap A_27a A_27b)} \end{aligned} \quad (11)$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EISL \\ & A_27a A_27b \in (2^{(ty_2Esum_2Esum A_27a A_27b)}) \end{aligned} \quad (12)$$

Definition 15 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFmap A_27a A_27b)$

Definition 16 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 17 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A-27a}). (ap V1f V0x)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & A_27a A_27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod A_27a 2)^{A-27b}}) \end{aligned} \quad (13)$$

Definition 18 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c_2Epred_set_2EINTER V0s V1t))$

Definition 19 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (c_2Ebool_2E_5C_2F V0t1 V2t))))))$

Definition 20 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A-27a}). (ap (c_2Epred_set_2EINSERT V0x V1s))$

Definition 21 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a.c_2Ebool_2E2ET)$.

Definition 22 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A-27a}). \lambda V1t \in (2^{A-27a}). (ap (c_2Epred_set_2EDIFF V0s V1t))$

Definition 23 We define $c_2Epred_set_2ECOMPL$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A-27a}). (ap (ap (c_2Epred_set_2ECOMPL V0P)))$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow \text{False}) \Rightarrow (\neg(p V0t)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow \text{False}))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge \text{True}) \Leftrightarrow \\ & (p V0t)) \wedge (((\text{False} \wedge (p V0t)) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \vee (p V0t)) \Leftrightarrow \text{True}) \wedge (((p V0t) \vee \text{True}) \Leftrightarrow \text{True}) \wedge \\ & (((\text{False} \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \text{False}) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow \text{True}) \Leftrightarrow \\ & \text{True}) \wedge (((\text{False} \Rightarrow (p V0t)) \Leftrightarrow \text{True}) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow \text{True}) \wedge ((\\ & (p V0t) \Rightarrow \text{False}) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge \\ & ((\neg \text{False}) \Leftrightarrow \text{True})))) \end{aligned} \quad (23)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((V0x = V0x) \Leftrightarrow \text{True})) \quad (24)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27a}.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow \text{True}) \Leftrightarrow \\ & (p V0t)) \wedge (((\text{False} \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ & (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ & (p\ V0A)))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ & p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0b \in 2. (\forall V1f \in (A_27b^{A_27a}). (\forall V2g \in (A_27b^{A_27a}). \\ & (\forall V3x \in A_27a. ((ap\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (A_27b^{A_27a})) \\ & V0b)\ V1f)\ V2g)\ V3x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V0b)\ (ap \\ & V1f\ V3x))\ (ap\ V2g\ V3x)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in 2. (\forall V2x \in A_27a. \\ & (\forall V3y \in A_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ V1b)\ (ap\ V0f \\ & V2x))\ (ap\ V0f\ V3y)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p\ (ap\ (ap \\ & (ap\ (c_2Ebool_2ECOND\ 2)\ V0b)\ V1t1)\ V2t2)) \Leftrightarrow (((\neg(p\ V0b)) \vee (p\ V1t1)) \wedge \\ & ((p\ V0b) \vee (p\ V2t2)))))) \end{aligned} \quad (34)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (35)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) V1Q) V3x_{.27}) \\ & V5y_{.27}))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\ & A_{.27a}.((ap (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2ET}} V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap \\ & (ap (ap (c_{.2Ebool_{.2ECOND}} A_{.27a}) c_{.2Ebool_{.2EF}} V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0f \in (ty_{.2Efinite_map_{.2E fmap}} A_{.27a} A_{.27b}).(\forall V1a \in \\ & A_{.27a}.(\forall V2b \in A_{.27b}.((ap (c_{.2Efinite_map_{.2EFDOM}} A_{.27a} \\ & A_{.27b}) (ap (ap (c_{.2Efinite_map_{.2EFUPDATE}} A_{.27a} A_{.27b}) V0f) (ap \\ & (ap (c_{.2Epair_{.2E_{.2C}} A_{.27a} A_{.27b}) V1a) V2b))) = (ap (ap (c_{.2Epred_set_{.2EINSERT}} \\ & A_{.27a}) V1a) (ap (c_{.2Efinite_map_{.2EFDOM}} A_{.27a} A_{.27b}) V0f)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0f \in (ty_{.2Efinite_map_{.2E fmap}} A_{.27a} A_{.27b}).(\forall V1a \in \\ & A_{.27a}.(\forall V2b \in A_{.27b}.(\forall V3x \in A_{.27a}.((ap (ap (c_{.2Efinite_map_{.2EFAPPLY}} \\ & A_{.27a} A_{.27b}) (ap (ap (c_{.2Efinite_map_{.2EFUPDATE}} A_{.27a} A_{.27b}) V0f) \\ & (ap (ap (c_{.2Epair_{.2E_{.2C}} A_{.27a} A_{.27b}) V1a) V2b))) V3x) = (ap (ap (ap \\ & (c_{.2Ebool_{.2ECOND}} A_{.27b}) (ap (ap (c_{.2Emin_{.2E_{.3D}} A_{.27a}) V3x) V1a)) \\ & V2b) (ap (ap (c_{.2Efinite_map_{.2EFAPPLY}} A_{.27a} A_{.27b}) V0f) V3x)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V1g \in \\
& \quad (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(((ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ V0f) = (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V1g)) \wedge \\
& \quad (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ (ap\ (\\
& \quad c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V0f)))) \Rightarrow ((ap\ (ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ V0f)\ V2x) = (ap\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a \\
& \quad A.27b)\ V1g)\ V2x)))))) \Leftrightarrow (V0f = V1g))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b).(\forall V1r \in \\
& \quad (2^{A.27a}).(((ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ (ap\ (ap \\
& \quad (c_2Efinite_map_2EDRESTRICT\ A.27a\ A.27b)\ V0f)\ V1r)) = (ap\ (ap \\
& \quad (c_2Epred_set_2EINTER\ A.27a)\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a \\
& \quad A.27b)\ V0f)\ V1r)) \wedge (\forall V2x \in A.27a.((ap\ (ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A.27a\ A.27b) \\
& \quad V0f)\ V1r))\ V2x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A.27b)\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27a)\ V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A.27a)\ (ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ V0f)\ V1r)))\ (ap\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a \\
& \quad A.27b)\ V0f)\ V2x))\ (ap\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b) \\
& \quad (c_2Efinite_map_2EFEMPTY\ A.27a\ A.27b))\ V2x))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\
& \quad (2^{A.27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\neg (p\ (ap\ (ap \\
& \quad (c_2Ebool_2EIN\ A.27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A.27a))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\
& \quad (2^{A.27a}).(\forall V2x \in A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a) \\
& \quad V2x)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ & (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2EINCOMPL \\ & A_27a)\ V1s))) \Leftrightarrow (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V1s)))))) \end{aligned} \quad (46)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (47)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (50)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (51)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r)))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q)) \Rightarrow (p V0p))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q)) \Rightarrow \neg(p V1q))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q)) \Rightarrow \neg(p V0p))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q)) \Rightarrow \neg(p V1q))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p)) \Rightarrow (p V0p))) \tag{61}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27b). (\forall V1r \in \\
& (2^{A_27a}). (\forall V2x \in A_27a. (\forall V3y \in A_27b. ((ap\ (ap\ (c_2Efinite_map_2EDRESTRICT \\
& A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V0f) \\
& (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y)))\ V1r) = (ap\ (ap\ (ap \\
& (c_2Ebool_2ECOND\ (ty_2Efinite_map_2E fmap\ A_27a\ A_27b))\ (ap \\
& (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1r))\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE \\
& A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b) \\
& V0f)\ (ap\ (ap\ (c_2Epred_set_2EINTER\ A_27a)\ (ap\ (c_2Epred_set_2EOMPL \\
& A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2x)\ (c_2Epred_set_2EEMPTY \\
& A_27a))))\ V1r)))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y))) \\
& (ap\ (ap\ (c_2Efinite_map_2EDRESTRICT\ A_27a\ A_27b)\ V0f)\ (ap\ (ap \\
& (c_2Epred_set_2EINTER\ A_27a)\ (ap\ (c_2Epred_set_2EOMPL\ A_27a) \\
& (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2x)\ (c_2Epred_set_2EEMPTY \\
& A_27a))))\ V1r))))))
\end{aligned}$$