

thm\_2Efinite\_\_map\_2ESUBMAP\_\_FUNION\_\_EQ  
(TMHGSmWM-  
FEvFf7dQ8iacbCCbacFGFieToyX)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$ .  
Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $ty\_2Efinite\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_map\_2Efmap\ A0\ A1) \tag{3}$$

Let  $c\_2Efinite\_map\_2Efmap\_REP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2Efmap\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)}) \tag{4}$$

Let  $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EOUTL\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{5}$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Efinite\_map\_2EFAPPLY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_map\_2Efmap$

Let  $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EISL \\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \end{aligned} \quad (6)$$

**Definition 5** We define  $c\_2Efinite\_map\_2EFDOM$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_map\_2E...$

**Definition 6** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(\text{ap } V1f\ V0x)))$

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(\text{ap } (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2...)))$

**Definition 9** We define  $c\_2Efinite\_map\_2ESUBMAP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_map\_2E...$

Let  $c\_2Efinite\_map\_2EFUNION : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2EFUNION \\ A\_27a\ A\_27b \in (((ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b)^{(ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_map\_2E fmap\ A\_27a\ A\_27b)}) \end{aligned} \quad (7)$$

**Definition 10** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(\text{ap } (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2...)))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod \\ A0\ A1) \end{aligned} \quad (8)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \end{aligned} \quad (9)$$

**Definition 11** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(\text{ap } (c\_2Epair\_2EABS\_prod\ x\ y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \end{aligned} \quad (10)$$

**Definition 12** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(\text{ap } (c\_2Epred\_set\_2EGSPEC\ s\ t))$

**Definition 13** We define  $c\_2Ebool\_2EF$  to be  $(\text{ap } (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 14** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 15** We define  $c\_2Epred\_set\_2EINTER$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(\text{ap } (c\_2Epred\_set\_2EGSPEC\ s\ t))$

**Definition 16** We define  $c\_2Epred\_set\_2EDISJOINT$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap$

**Definition 17** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E$

**Definition 18** We define  $c\_2Epred\_set\_2EDIFF$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap (c\_2E$

**Definition 19** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A \wedge$   
of type  $\iota \Rightarrow \iota$ .

**Definition 20** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

Assume the following.

$$True \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \tag{14}$$

Assume the following.

$$((\forall V0t \in 2. ((\neg (p V0t)) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{15}$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (V0x = V0x)) \tag{16}$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg (p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg (p V0t)))))) \tag{18}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg ((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg (p V0A) \vee \neg (p V1B)))) \wedge ((\neg ((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg (p V0A) \wedge \neg (p V1B)))))) \tag{19}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.((((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A))))))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in 2.(\forall V2x \in A\_27a. \\ (\forall V3y \in A\_27a.((ap\ V0f\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V1b)\ (ap\ V0f\ \\ V2x))\ (ap\ V0f\ V3y))))))) \end{aligned} \quad (22)$$

Assume the following.

$$2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (ty\_2Efinite\_map\_2Efmmap\ A\_27a\ A\_27b).(\forall V1g \in \\ (ty\_2Efinite\_map\_2Efmmap\ A\_27a\ A\_27b).(((ap\ (c\_2Efinite\_map\_2EFDOM \\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUNION\ A\_27a\ A\_27b)\ V0f) \\ V1g)) = (ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a)\ (ap\ (c\_2Efinite\_map\_2EFDOM \\ A\_27a\ A\_27b)\ V0f))\ (ap\ (c\_2Efinite\_map\_2EFDOM\ A\_27a\ A\_27b)\ V1g))) \wedge \\ (\forall V2x \in A\_27a.((ap\ (ap\ (c\_2Efinite\_map\_2EFAPPLY\ A\_27a \\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUNION\ A\_27a\ A\_27b)\ V0f)\ V1g)) \\ V2x) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ A\_27a)\ V2x)\ (ap\ (c\_2Efinite\_map\_2EFDOM\ A\_27a\ A\_27b)\ V0f)))\ (ap \\ (ap\ (c\_2Efinite\_map\_2EFAPPLY\ A\_27a\ A\_27b)\ V0f)\ V2x))\ (ap\ (ap\ ( \\ c\_2Efinite\_map\_2EFAPPLY\ A\_27a\ A\_27b)\ V1g)\ V2x)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t)))))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a)))) \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ (2^{A\_27a}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EUNION\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ ( \\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ A\_27a)\ V2x)\ V1t)))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ (2^{A\_27a}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINTER\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ ( \\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \wedge (p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ A\_27a)\ V2x)\ V1t)))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). (\forall V1t \in \\ (2^{A\_27a}). (\forall V2x \in A\_27a. ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\ V2x)\ (ap\ (ap\ (c\_2Epred\_set\_2EDIFF\ A\_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ ( \\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V0s)) \wedge (\neg(p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\ A\_27a)\ V2x)\ V1t)))))))) \end{aligned} \quad (29)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg( \\ p\ V2r)) \vee (\neg(p\ V1q))) \wedge ((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge ( \\
& \neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (\forall V3s \in \\
& 2. (((p V0p) \Leftrightarrow (p (ap (ap (c_2Ebool_2ECOND 2) V1q) V2r) V3s))) \Leftrightarrow \\
& (((p V0p) \vee ((p V1q) \vee (\neg(p V3s)))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge \\
& (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V3s)))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg \\
& p V0p)))) \wedge ((p V1q) \vee ((p V3s) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{42}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{43}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{44}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{45}$$

**Theorem 1**

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\
& \quad nonempty\ A_{.27c} \Rightarrow \forall A_{.27d}.nonempty\ A_{.27d} \Rightarrow ((\forall V0f1 \in \\
& (ty\_2Efinite\_map\_2Efmap\ A_{.27a}\ A_{.27b}).(\forall V1f2 \in (ty\_2Efinite\_map\_2Efmap \\
& \quad A_{.27a}\ A_{.27b}).(\forall V2f3 \in (ty\_2Efinite\_map\_2Efmap\ A_{.27a}\ A_{.27b}). \\
& \quad ((p\ (ap\ (ap\ (c\_2Epred\_set\_2EDISJOINT\ A_{.27a})\ (ap\ (c\_2Efinite\_map\_2EFDOM \\
& \quad A_{.27a}\ A_{.27b})\ V0f1))\ (ap\ (c\_2Efinite\_map\_2EFDOM\ A_{.27a}\ A_{.27b})\ V1f2))) \Rightarrow \\
& \quad ((p\ (ap\ (ap\ (c\_2Efinite\_map\_2ESUBMAP\ A_{.27a}\ A_{.27b})\ V0f1)\ (ap\ (ap \\
& \quad (c\_2Efinite\_map\_2EFUNION\ A_{.27a}\ A_{.27b})\ V1f2)\ V2f3))) \Leftrightarrow (p\ (ap\ ( \\
& \quad ap\ (c\_2Efinite\_map\_2ESUBMAP\ A_{.27a}\ A_{.27b})\ V0f1)\ V2f3)))))) \wedge \\
& \quad (\forall V3f1 \in (ty\_2Efinite\_map\_2Efmap\ A_{.27c}\ A_{.27d}).(\forall V4f2 \in \\
& (ty\_2Efinite\_map\_2Efmap\ A_{.27c}\ A_{.27d}).(\forall V5f3 \in (ty\_2Efinite\_map\_2Efmap \\
& \quad A_{.27c}\ A_{.27d}).((p\ (ap\ (ap\ (c\_2Epred\_set\_2EDISJOINT\ A_{.27c})\ (ap \\
& \quad (c\_2Efinite\_map\_2EFDOM\ A_{.27c}\ A_{.27d})\ V3f1))\ (ap\ (ap\ (c\_2Epred\_set\_2EDIFF \\
& \quad A_{.27c})\ (ap\ (c\_2Efinite\_map\_2EFDOM\ A_{.27c}\ A_{.27d})\ V5f3))\ (ap\ (c\_2Efinite\_map\_2EFDOM \\
& \quad A_{.27c}\ A_{.27d})\ V4f2)))))) \Rightarrow ((p\ (ap\ (ap\ (c\_2Efinite\_map\_2ESUBMAP\ A_{.27c} \\
& \quad A_{.27d})\ V3f1)\ (ap\ (ap\ (c\_2Efinite\_map\_2EFUNION\ A_{.27c}\ A_{.27d})\ V4f2) \\
& \quad V5f3))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Efinite\_map\_2ESUBMAP\ A_{.27c}\ A_{.27d})\ V3f1) \\
& \quad V4f2)))))))))
\end{aligned}$$