

thm_2Efinite_map_2ESUBMAP_FUPDATE_FLOOKUP (TMascb5e4nPznRgnu2HyYbcndSiMNtoTF6Q)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (2)$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod A_27a A_27b) (ty_2Epair_2Eprod A_27a A_27b))$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \quad (3)$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b \in (((ty_2Efinite_map_2Efm\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)})$$
(4)

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone$$
(5)

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1)$$
(6)

Let $c_2Efinite_map_2Efm_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efm_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efm\ A_27a\ A_27b)})$$
(7)

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a)^{(ty_2Esum_2Esum\ A_27a\ A_27b)}$$
(8)

Definition 8 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)})$$
(9)

Definition 9 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Definition 11 We define $c_2Efinite_map_2ESUBMAP$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. P\ x) \mathbf{then} (the\ (\lambda x. x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone. V0x))$

Definition 14 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2})^{(ty_2Esum_2Esum\ A_27a\ A_27b)})$$
(10)

Definition 16 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (11)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (12)$$

Definition 17 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c$

Definition 18 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS$

Definition 19 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS$

Definition 20 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 21 We define $c_2Efinite_map_2EFLOOKUP$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Definition 22 We define $c_2Emarker_2EAC$ to be $\lambda V0b1 \in 2. \lambda V1b2 \in 2. (ap (ap c_2Ebool_2E_2F_5C V0b1) ($

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \wedge (p V1t2)) \Leftrightarrow ((p V1t2) \wedge (p V0t1)))))) \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (15)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\ & \quad \forall V0f \in (ty_2Efinite_map_2Efmap A_27a A_27b). (\forall V1x \in \\ & \quad A_27a. (\forall V2y \in A_27b. ((p (ap (ap (c_2Efinite_map_2ESUBMAP \\ & \quad A_27a A_27b) V0f) (ap (ap (c_2Efinite_map_2EFUPDATE A_27a A_27b) \\ & \quad V0f) (ap (ap (c_2Epair_2E_2C A_27a A_27b) V1x) V2y)))))) \Leftrightarrow ((\neg (p (ap \\ & \quad (ap (c_2Ebool_2EIN A_27a) V1x) (ap (c_2Efinite_map_2EFDOM A_27a \\ & \quad A_27b) V0f)))) \vee (((ap (ap (c_2Efinite_map_2EFAPPLY A_27a A_27b) \\ & \quad V0f) V1x) = V2y) \wedge (p (ap (ap (c_2Ebool_2EIN A_27a) V1x) (ap (c_2Efinite_map_2EFDOM \\ & \quad A_27a A_27b) V0f)))))))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1x \in A.27a. \\
& (\forall V2y \in A.27a. (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption \\
& A.27a))\ V0P)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x))\ (c.2Eoption.2ENONE \\
& A.27a)) = (c.2Eoption.2ENONE\ A.27a)) \Leftrightarrow (\neg(p\ V0P))) \wedge (((ap\ (ap\ (\\
& ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ A.27a))\ V0P)\ (c.2Eoption.2ENONE \\
& A.27a))\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x)) = (c.2Eoption.2ENONE \\
& A.27a)) \Leftrightarrow (p\ V0P)) \wedge (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption \\
& A.27a))\ V0P)\ (ap\ (c.2Eoption.2ESOME\ A.27a)\ V1x))\ (c.2Eoption.2ENONE \\
& A.27a)) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge \\
& (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ (ty.2Eoption.2Eoption\ A.27a)) \\
& V0P)\ (c.2Eoption.2ENONE\ A.27a))\ (ap\ (c.2Eoption.2ESOME\ A.27a) \\
& V1x)) = (ap\ (c.2Eoption.2ESOME\ A.27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = \\
& V2y)))))))))
\end{aligned} \tag{18}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (ty.2Efinite_map.2Efmap\ A.27a\ A.27b). (\forall V1x \in \\
& A.27a. (\forall V2y \in A.27b. ((p\ (ap\ (ap\ (c.2Efinite_map.2ESUBMAP \\
& A.27a\ A.27b)\ V0f)\ (ap\ (ap\ (c.2Efinite_map.2EFUPDATE\ A.27a\ A.27b) \\
& V0f)\ (ap\ (ap\ (c.2Epair.2E2C\ A.27a\ A.27b)\ V1x)\ V2y)))))) \Leftrightarrow (((ap\ (ap \\
& (c.2Efinite_map.2EFLOOKUP\ A.27a\ A.27b)\ V0f)\ V1x) = (c.2Eoption.2ENONE \\
& A.27b)) \vee ((ap\ (ap\ (c.2Efinite_map.2EFLOOKUP\ A.27a\ A.27b)\ V0f) \\
& V1x) = (ap\ (c.2Eoption.2ESOME\ A.27b)\ V2y))))))
\end{aligned}$$