

# thm\_2Efinite\_map\_2Efmap\_EXT (TMWR- MJVN3GsTVw2T75samt6qNDe7uMmP89P)

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Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $ty\_2Efinite\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_map\_2Efmap\ A0\ A1) \tag{3}$$

Let  $c\_2Efinite\_map\_2Efmap\_REP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_map\_2Efmap\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)}) \tag{4}$$

Let  $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EOUTL\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{5}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E21$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A\_27a}))\ (\lambda V1f \in V0f))))$

**Definition 4** We define  $c\_2Efinite\_map\_2EFAPPLY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_map\_2Efmap\ A\_27a\ A\_27b)$

**Definition 5** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EISL \\ A\_27a A\_27b \in (2^{(ty\_2Esum\_2Esum A\_27a A\_27b)}) \end{aligned} \quad (6)$$

**Definition 7** We define  $c\_2Efinite\_map\_2EFDOM$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_map\_2EFDOM A\_27a A\_27b)$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.(ap (c\_2Ebool\_2E\_21 2) V2t))))))$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ \forall V0f \in (ty\_2Efinite\_map\_2Efdom A\_27a A\_27b).(\forall V1g \in \\ (ty\_2Efinite\_map\_2Efdom A\_27a A\_27b).(((ap (c\_2Efinite\_map\_2EFDOM \\ A\_27a A\_27b) V0f) = (ap (c\_2Efinite\_map\_2EFDOM A\_27a A\_27b) V1g))) \wedge \\ (\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) (ap ( \\ c\_2Efinite\_map\_2EFDOM A\_27a A\_27b) V0f)))) \Rightarrow ((ap (ap (c\_2Efinite\_map\_2EFAPPLY \\ A\_27a A\_27b) V0f) V2x) = (ap (ap (c\_2Efinite\_map\_2EFAPPLY A\_27a \\ A\_27b) V1g) V2x)))))) \Leftrightarrow (V0f = V1g)))) \end{aligned} \quad (8)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ \forall V0f \in (ty\_2Efinite\_map\_2Efdom A\_27a A\_27b).(\forall V1g \in \\ (ty\_2Efinite\_map\_2Efdom A\_27a A\_27b).((V0f = V1g) \Leftrightarrow (((ap (c\_2Efinite\_map\_2EFDOM \\ A\_27a A\_27b) V0f) = (ap (c\_2Efinite\_map\_2EFDOM A\_27a A\_27b) V1g))) \wedge \\ (\forall V2x \in A\_27a.((p (ap (ap (c\_2Ebool\_2EIN A\_27a) V2x) (ap ( \\ c\_2Efinite\_map\_2EFDOM A\_27a A\_27b) V0f)))) \Rightarrow ((ap (ap (c\_2Efinite\_map\_2EFAPPLY \\ A\_27a A\_27b) V0f) V2x) = (ap (ap (c\_2Efinite\_map\_2EFAPPLY A\_27a \\ A\_27b) V1g) V2x))))))))) \end{aligned}$$