

thm_2Efinite__map_2Efmap__SIMPLE__INDUCT (TMb9fZKLExRyZXG61tcJEcCSwwAHXHjiY5x)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2EF$

Definition 7 We define $c_2Emin_2E_240$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Ebool_2E_23F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_240 A$

Definition 9 We define $c_2Ebool_2E_25C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 10 We define $c_2Ebool_2E_25C_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A-27b})^{A-27a})^2}) \tag{3}$$

Definition 11 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS$

Definition 12 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 13 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2E$

Definition 14 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$

Definition 15 We define $c_2Efinite_map_2Eis_fmap$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0a0 \in ((ty_2Esum_2E$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \quad (4)$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_REP A_27a A_27b \in (((ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap A_27a A_27b)}) \quad (5)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EFUPDATE A_27a A_27b \in (((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)})^{(ty_2Efinite_map_2EFUPDATE A_27a A_27b)}) \quad (8)$$

Let $c_2Efinite_map_2Efmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_ABS A_27a A_27b \in ((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a}}) \quad (9)$$

Definition 17 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Efinite_map_2E$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w)) \Rightarrow (((p V0x) \wedge (p V2z)) \Rightarrow ((p V1y) \wedge (p V3w))))))) \quad (15)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p V0x) \Rightarrow (p V1y)) \wedge ((p V2z) \Rightarrow (p V3w)) \Rightarrow (((p V0x) \vee (p V2z)) \Rightarrow ((p V1y) \vee (p V3w))))))) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V0P V2x)) \Rightarrow (p (ap V1Q V2x)))) \Rightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x))) \Rightarrow (\exists V4x \in A.27a.(p (ap V1Q V4x))))))) \quad (17)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow ((\forall V0a \in (ty_2Efinite_map_2E fmap A.27a A.27b).((ap (c.2Efinite_map_2E fmap_ABS A.27a A.27b) (ap (c.2Efinite_map_2E fmap_REP A.27a A.27b) V0a)) = V0a)) \wedge (\forall V1r \in ((ty_2Esum_2Esum A.27b ty_2Eone_2Eone)^{A.27a}).((p (ap (c.2Efinite_map_2Eis_fmap A.27a A.27b) V1r)) \Leftrightarrow ((ap (c.2Efinite_map_2E fmap_REP A.27a A.27b) (ap (c.2Efinite_map_2E fmap_ABS A.27a A.27b) V1r)) = V1r)))) \quad (18)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27b). (\forall V1x \in \\
& \quad A_27a. (\forall V2y \in A_27b. ((ap\ (ap\ (c_2Efinite_map_2EFUPDATE \\
& \quad A_27a\ A_27b)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y)) = \\
& \quad (ap\ (c_2Efinite_map_2E fmap_ABS\ A_27a\ A_27b)\ (\lambda V3a \in A_27a. \\
& \quad (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)) \\
& \quad (ap\ (ap\ (c_2Emin_2E_3D\ A_27a)\ V3a)\ V1x))\ (ap\ (c_2Esum_2EINL\ A_27b \\
& \quad ty_2Eone_2Eone)\ V2y))\ (ap\ (ap\ (c_2Efinite_map_2E fmap_REP\ A_27a \\
& \quad A_27b)\ V0f)\ V3a))))))))) \\
& \hspace{15em} (19)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in (2^{(ty_2Efinite_map_2E fmap\ A_27a\ A_27b)}). (((p \\
& \quad (ap\ V0P\ (c_2Efinite_map_2EFEMPTY\ A_27a\ A_27b))) \wedge (\forall V1f \in \\
& \quad (ty_2Efinite_map_2E fmap\ A_27a\ A_27b). ((p\ (ap\ V0P\ V1f)) \Rightarrow (\forall V2x \in \\
& \quad A_27a. (\forall V3y \in A_27b. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE \\
& \quad A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2x)\ V3y))))))))) \Rightarrow \\
& \quad (\forall V4f \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27b). (p\ (ap\ V0P \\
& \quad V4f))))))
\end{aligned}$$