

thm_2Efinite__map_2Efmap__rel__FUNION__rels (TMU63vjz3TaDeRgG7xKnhB58FWFV1ejhtqR)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Ebool_2E_COND$ to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \tag{1}$$

Let $c_2Efinite_map_2EFUNION : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EFUNION A_27a A_27b \in (((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Efinite_map_2Efmap A_27a A_27b)})^{(ty_2Efinite_map_2Efmap A_27a A_27b)})^{(ty_2Efinite_map_2Efmap A_27a A_27b)} \tag{2}$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty ty_2Eone_2Eone \tag{3}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (4)$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \quad (5)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a)^{(ty_2Esum_2Esum\ A_27a\ A_27b)} \quad (6)$$

Definition 10 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFmap_REP\ A_27a\ A_27b).V0f$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (7)$$

Definition 11 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFmap_REP\ A_27a\ A_27b).V0f$

Definition 12 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 13 We define $c_2Efinite_map_2Efmap_rel$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0R \in ((2^{A_27a})^{(2^{A_27b})})^{A_27c}$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (8)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (9)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2z \in 2.V2z)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (10)$$

Definition 16 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in A_27a.(((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))))) \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (20)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \Rightarrow (21)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A.27a.(\forall V3x.27 \in A.27a.(\forall V4y \in A.27a. \\ & (\forall V5y.27 \in A.27a.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x.27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y.27)))) \Rightarrow ((ap (ap (ap (c.2Ebool.2ECOND A.27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c.2Ebool.2ECOND A.27a) V1Q) V3x.27) \\ & V5y.27))))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow ((\forall V0t1 \in A.27a.(\forall V1t2 \in \\ & A.27a.((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A.27a.(\forall V3t2 \in A.27a.((ap \\ & (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF) V2t1) V3t2) = V3t2)))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow (\\ & \forall V0f \in (ty.2Efinite_map.2Efmap A.27a A.27b).(\forall V1g \in \\ & (ty.2Efinite_map.2Efmap A.27a A.27b).(((ap (c.2Efinite_map.2EFDOM \\ & A.27a A.27b) (ap (ap (c.2Efinite_map.2EFUNION A.27a A.27b) V0f) \\ & V1g)) = (ap (ap (c.2Epred_set.2EUNION A.27a) (ap (c.2Efinite_map.2EFDOM \\ & A.27a A.27b) V0f)) (ap (c.2Efinite_map.2EFDOM A.27a A.27b) V1g)))) \wedge \\ & (\forall V2x \in A.27a.((ap (ap (c.2Efinite_map.2EFAPPLY A.27a \\ & A.27b) (ap (ap (c.2Efinite_map.2EFUNION A.27a A.27b) V0f) V1g)) \\ & V2x) = (ap (ap (ap (c.2Ebool.2ECOND A.27b) (ap (ap (c.2Ebool.2EIN \\ & A.27a) V2x) (ap (c.2Efinite_map.2EFDOM A.27a A.27b) V0f))) (ap \\ & (ap (c.2Efinite_map.2EFAPPLY A.27a A.27b) V0f) V2x)) (ap (ap (\\ & c.2Efinite_map.2EFAPPLY A.27a A.27b) V1g) V2x)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(\forall V1t \in \\ & (2^{A.27a}).(\forall V2x \in A.27a.((p (ap (ap (c.2Ebool.2EIN A.27a) \\ & V2x) (ap (ap (c.2Epred_set.2EUNION A.27a) V0s) V1t))) \Leftrightarrow ((p (ap \\ & (ap (c.2Ebool.2EIN A.27a) V2x) V0s)) \vee (p (ap (ap (c.2Ebool.2EIN \\ & A.27a) V2x) V1t)))))) \end{aligned} \quad (25)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0R \in ((2^{A_27b})^{A_27a}). (\forall V1f1 \in \\ & (ty_2Efinite_map_2E fmap\ A_27c\ A_27a). (\forall V2f2 \in (ty_2Efinite_map_2E fmap \\ & \quad A_27c\ A_27b). (\forall V3f3 \in (ty_2Efinite_map_2E fmap\ A_27c\ A_27a). \\ & \quad (\forall V4f4 \in (ty_2Efinite_map_2E fmap\ A_27c\ A_27b). (((p\ (ap \\ & \quad (ap\ (ap\ (c_2Efinite_map_2E fmap_rel\ A_27a\ A_27b\ A_27c)\ V0R)\ V1f1) \\ & \quad V2f2)) \wedge (p\ (ap\ (ap\ (ap\ (c_2Efinite_map_2E fmap_rel\ A_27a\ A_27b \\ & \quad A_27c)\ V0R)\ V3f3)\ V4f4))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Efinite_map_2E fmap_rel \\ & \quad A_27a\ A_27b\ A_27c)\ V0R)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION\ A_27c \\ & \quad A_27a)\ V1f1)\ V3f3))\ (ap\ (ap\ (c_2Efinite_map_2EFUNION\ A_27c\ A_27b) \\ & \quad V2f2)\ V4f4)))))))))) \end{aligned}$$