

Definition 11 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2Efmap\ A0\ A1) \quad (4)$$

Let $c_2Efinite_map_2Efmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_ABS\ A_27a\ A_27b \in ((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a}}) \quad (5)$$

Definition 12 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Efinite_map_2E$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b \in (((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2E}} \quad (7)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (8)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL\ A_27a\ A_27b \in (((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b^{A_27a})^{A_27b})}) \quad (9)$$

Definition 13 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Elist_2E$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \quad (10)$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Esum_2EISL\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (11)$$

Definition 14 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ & A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b^{A_27a})}) \end{aligned} \quad (12)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in \\ & ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \end{aligned} \quad (13)$$

Definition 15 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist \\ & A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \end{aligned} \quad (14)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist \\ & A_27a) \end{aligned} \quad (15)$$

Let $c_2Elist_2EALL_DISTINCT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EALL_DISTINCT\ A_27a \in \\ & (2^{(ty_2Elist_2Elist\ A_27a)}) \end{aligned} \quad (16)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ & A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})}) \end{aligned} \quad (17)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Ebool_2EIN\ V0x)\ V1y)$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ & A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (18)$$

Definition 17 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (c_2Ebool_2E_5C_2F\ V2t)\ V1t2))\ V0t1))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (19)$$

Definition 19 We define `c_2Epred_set_2EUNION` to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Efinite_map_2EFDOM A_27a A_27b) (c_2Efinite_map_2EFEMPTY A_27a A_27b)) = (c_2Epred_set_2EEMPTY A_27a)$. Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (23)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (24)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (26)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (ap (c_2Efinite_map_2EFDOM A_27a A_27b) (c_2Efinite_map_2EFEMPTY A_27a A_27b)) = (c_2Epred_set_2EEMPTY A_27a) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\ & \quad \forall V0P \in (2^{(ty_2Efinite_map_2Emap A_27a A_27b)}). (((p \\ & \quad (ap V0P (c_2Efinite_map_2EFEMPTY A_27a A_27b))) \wedge (\forall V1f \in \\ & \quad (ty_2Efinite_map_2Emap A_27a A_27b). ((p (ap V0P V1f)) \Rightarrow (\forall V2x \in \\ & \quad A_27a. (\forall V3y \in A_27b. ((\neg(p (ap (ap (c_2Ebool_2EIN A_27a) \\ & \quad V2x) (ap (c_2Efinite_map_2EFDOM A_27a A_27b) V1f)))) \Rightarrow (p (ap V0P \\ & \quad (ap (ap (c_2Efinite_map_2EFUPDATE A_27a A_27b) V1f) (ap (ap (c_2Epair_2E_2C \\ & \quad A_27a A_27b) V2x) V3y)))))))))) \Rightarrow (\forall V4f \in (ty_2Efinite_map_2Emap \\ & \quad A_27a A_27b). (p (ap V0P V4f)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2Efmmap\ A_27a\ A_27b).((ap\ (ap \\
& \quad (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b)\ V0f)\ (c_2Elist_2ENIL \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b)))) = V0f) \wedge (\forall V1h \in (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b).(\forall V2t \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)).((ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a \\
& \quad A_27b)\ V0f)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ A_27a\ A_27b)) \\
& \quad V1h)\ V2t)) = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b) \\
& \quad (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V0f)\ V1h))\ V2t)))))) \\
& \hspace{15em} (29)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0k \in A_27a.(\forall V1kvl \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod \\
& \quad A_27a\ A_27b)).(\forall V2fm \in (ty_2Efinite_map_2Efmmap\ A_27a \\
& \quad A_27b).(\forall V3v \in A_27b.((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& \quad V0k)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap\ (ap\ (c_2Elist_2EMAP \\
& \quad (ty_2Epair_2Eprod\ A_27a\ A_27b)\ A_27a)\ (c_2Epair_2EFST\ A_27a\ A_27b)) \\
& \quad V1kvl)))))) \Rightarrow ((ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a \\
& \quad A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ V2fm)\ (\\
& \quad ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0k)\ V3v)))\ V1kvl) = (ap\ (ap\ (\\
& \quad c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST \\
& \quad A_27a\ A_27b)\ V2fm)\ V1kvl))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\
& \quad V0k)\ V3v)))))) \\
& \hspace{15em} (30)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0kvl \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). \\
& \quad (\forall V1fm \in (ty_2Efinite_map_2Efmmap\ A_27a\ A_27b).((ap\ (c_2Efinite_map_2EFDOM \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b) \\
& \quad V1fm)\ V0kvl)) = (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ (ap\ (c_2Efinite_map_2EFDOM \\
& \quad A_27a\ A_27b)\ V1fm))\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (ap\ (\\
& \quad ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_27a\ A_27b)\ A_27a)\ (c_2Epair_2EFST \\
& \quad A_27a\ A_27b))\ V0kvl)))))) \\
& \hspace{15em} (31)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad (\forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b) \\
& \quad V0f)\ (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27b))) \wedge (\forall V1f \in \\
& \quad (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist \\
& \quad A_27a).((ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ (ap\ V1f\ V2h)) \\
& \quad (ap\ (ap\ (c_2Elist_2EMAP\ A_27a\ A_27b)\ V1f)\ V3t)))))) \\
& \hspace{15em} (32)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (((p\ (ap\ (c_2Elist_2EALL_DISTINCT \\ & A_27a)\ (c_2Elist_2ENIL\ A_27a))) \Leftrightarrow True) \wedge (\forall V0h \in A_27a. (\\ \forall V1t \in (ty_2Elist_2Elist\ A_27a). & ((p\ (ap\ (c_2Elist_2EALL_DISTINCT \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a\ V0h)\ V1t))) \Leftrightarrow ((\neg(p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a) \\ & V1t)))) \wedge (p\ (ap\ (c_2Elist_2EALL_DISTINCT\ A_27a\ V1t))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a. (\forall V1y \in A_27b. & ((ap\ (c_2Epair_2EFST\ A_27a \\ & A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0s \in (2^{A_27a}). ((ap\ (\\ ap\ (c_2Epred_set_2EUNION\ A_27a)\ & (c_2Epred_set_2EEMPTY\ A_27a)) \\ V0s) = V0s)) \wedge (\forall V1s \in & (2^{A_27a}). ((ap\ (ap\ (c_2Epred_set_2EUNION \\ & A_27a)\ V1s)\ (c_2Epred_set_2EEMPTY\ A_27a)) = V1s))) \end{aligned} \quad (35)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0m \in (ty_2Efinite_map_2Efmap\ & A_27a\ A_27b). (\exists V1l \in \\ (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ & A_27a\ A_27b)). ((p\ (ap\ (c_2Elist_2EALL_DISTINCT \\ & A_27a)\ (ap\ (ap\ (c_2Elist_2EMAP\ (ty_2Epair_2Eprod\ A_27a\ A_27b) \\ & A_27a)\ (c_2Epair_2EFST\ A_27a\ A_27b))\ V1l))) \wedge (V0m = (ap\ (ap\ (c_2Efinite_map_2EFUPDATE_LIST \\ & A_27a\ A_27b)\ (c_2Efinite_map_2EFEMPTY\ A_27a\ A_27b))\ V1l)))) \end{aligned}$$