

thm_2Efinite__map_2Eis__fmap__strongind (TMF4jwUnTK9ooBDs98imPcKAbCo1Z32hxa)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_7E` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2E_7E` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (P \Rightarrow Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t)) (\text{c_2Ebool_2E_7E } V0t)))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (ty_2Esum_2Esum \ A0 \ A1) \tag{1}$$

Let `c_2Esum_2EABS__sum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow c_2Esum_2EABS_sum \ A_27a \ A_27b \in ((ty_2Esum_2Esum \ A_27a \ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{2}$$

Definition 8 We define `c_2Esum_2EINL` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (\text{ap } (\text{c_2Esum_2EABS_sum } V0e))$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P \ x) \text{ then } (the \ (\lambda x. x \in A \wedge P \ x))$ of type $\iota \Rightarrow \iota$.

Definition 10 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V2t2 \in A_27a))))$

Definition 11 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } V0P \ (\text{ap } (\text{c_2Emin_2E_40 } V0P))))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (3)$$

Definition 12 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS$

Definition 14 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 15 We define $c_2Efinite_map_2Eis_fmap$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0a0 \in ((ty_2Esum_2E$

Assume the following.

$$True \quad (4)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (5)$$

Assume the following.

$$(\forall V0t \in 2.(((True) \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))) \quad (6)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))) \quad (7)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.(((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \vee (p\ V2z)) \Rightarrow ((p\ V1y) \vee (p\ V3w)))))) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A_27a.(p\ (ap\ V1Q\ V4x)))))) \quad (9)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0is_fmap_27 \in (2^{((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})}), \\ & (((p\ (ap\ V0is_fmap_27\ (\lambda V1a \in A_27a.(ap\ (c_2Esum_2EINR\ A_27b \\ & ty_2Eone_2Eone)\ c_2Eone_2Eone)))) \wedge (\forall V2f \in ((ty_2Esum_2Esum \\ & A_27b\ ty_2Eone_2Eone)^{A_27a}).(\forall V3a \in A_27a.(\forall V4b \in \\ & A_27b.(((p\ (ap\ (c_2Efinite_map_2Eis_fmap\ A_27a\ A_27b)\ V2f)) \wedge \\ & (p\ (ap\ V0is_fmap_27\ V2f))) \Rightarrow (p\ (ap\ V0is_fmap_27\ (\lambda V5x \in A_27a. \\ & (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)) \\ & (ap\ (ap\ (c_2Emin_2E3D\ A_27a)\ V5x)\ V3a))\ (ap\ (c_2Esum_2EINL\ A_27b \\ & ty_2Eone_2Eone)\ V4b))\ (ap\ V2f\ V5x)))))))))) \Rightarrow (\forall V6a0 \in ((\\ & ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a}).((p\ (ap\ (c_2Efinite_map_2Eis_fmap \\ & A_27a\ A_27b)\ V6a0)) \Rightarrow (p\ (ap\ V0is_fmap_27\ V6a0)))))) \end{aligned}$$