

thm_2Efinite_map_2Eo_f_FUNION

(TMFMmY67p5jHmjTFEgE1G5r6orTN78Zk4ps)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 9 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \tag{1}$$

Let $c_2Efinite_map_2EFUNION : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EFUNION A_27a A_27b \in (((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Efinite_map_2Efmap A_27a A_27b)})^{(ty_2Efinite_map_2Efmap A_27a A_27b)})^{(ty_2Efinite_map_2Efmap A_27a A_27b)} \tag{2}$$

Let $c_2Efinite_map_2Eo_f : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c.nonempty A_27c \Rightarrow c_2Efinite_map_2Eo_f A_27a A_27b A_27c \in (((ty_2Efinite_map_2Efmap A_27a A_27c)^{(ty_2Efinite_map_2Efmap A_27a A_27c)})^{(A_27c^{A_27b})})^{(ty_2Efinite_map_2Efmap A_27a A_27b)} \tag{3}$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (4)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (5)$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)} \quad (6)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a)^{(ty_2Esum_2Esum\ A_27a\ A_27b)} \quad (7)$$

Definition 10 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFmap\ A_27a\ A_27b).$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL\ A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (8)$$

Definition 11 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map_2EFmap\ A_27a\ A_27b).$

Definition 12 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 13 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 14 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ (V0x\ V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (11)$$

Definition 15 We define $c_2\text{Epred_set_2EUNION}$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(\text{ap } (c$

Assume the following.

$$\text{True} \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg(p \ V0t)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t1 \in A.27a. (\forall V1t2 \in \\ & A.27a. (((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A.27a. (\forall V3x.27 \in A.27a. (\forall V4y \in A.27a. \\ & (\forall V5y.27 \in A.27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x.27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y.27)))) \Rightarrow ((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ V1Q)\ V3x.27) \\ & V5y.27)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0t1 \in A.27a. (\forall V1t2 \in \\ & A.27a. ((ap\ (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A.27a. (\forall V3t2 \in A.27a. ((ap \\ & (ap\ (ap\ (c.2Ebool.2ECOND\ A.27a)\ c.2Ebool.2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0f \in (ty.2Efinite_map.2Efmap\ A.27a\ A.27b). (\forall V1g \in \\ & (ty.2Efinite_map.2Efmap\ A.27a\ A.27b). (((ap\ (c.2Efinite_map.2EFDOM \\ & A.27a\ A.27b)\ V0f) = (ap\ (c.2Efinite_map.2EFDOM\ A.27a\ A.27b)\ V1g)) \wedge \\ & (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ (ap\ (\\ & c.2Efinite_map.2EFDOM\ A.27a\ A.27b)\ V0f))) \Rightarrow ((ap\ (ap\ (c.2Efinite_map.2EFAPPLY \\ & A.27a\ A.27b)\ V0f)\ V2x) = (ap\ (ap\ (c.2Efinite_map.2EFAPPLY\ A.27a \\ & A.27b)\ V1g)\ V2x)))) \Leftrightarrow (V0f = V1g)) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (ty_2Efinite_map_2E fmap\ A.27a\ A.27b). (\forall V1g \in \\
& \quad (ty_2Efinite_map_2E fmap\ A.27a\ A.27b). (((ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION\ A.27a\ A.27b)\ V0f) \\
& \quad V1g)) = (ap\ (ap\ (c_2Epred_set_2EUNION\ A.27a)\ (ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ V0f))\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V1g)))) \wedge \\
& \quad (\forall V2x \in A.27a. ((ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A.27a \\
& \quad A.27b)\ (ap\ (ap\ (c_2Efinite_map_2EFUNION\ A.27a\ A.27b)\ V0f)\ V1g))\ V2x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A.27b)\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27a)\ V2x)\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V0f)))\ (ap \\
& \quad (ap\ (c_2Efinite_map_2EFAPPLY\ A.27a\ A.27b)\ V0f)\ V2x))\ (ap\ (ap\ (\\
& \quad c_2Efinite_map_2EFAPPLY\ A.27a\ A.27b)\ V1g)\ V2x))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27c^{A.27b}). (\forall V1g \in (ty_2Efinite_map_2E fmap \\
& \quad A.27a\ A.27b). ((ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27c)\ (ap\ (ap \\
& \quad (c_2Efinite_map_2Eo_f\ A.27a\ A.27b\ A.27c)\ V0f)\ V1g)) = (ap\ (c_2Efinite_map_2EFDOM \\
& \quad A.27a\ A.27b)\ V1g))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0f \in (A.27c^{A.27b}). (\forall V1g \in (ty_2Efinite_map_2E fmap \\
& \quad A.27a\ A.27b). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a) \\
& \quad V2x)\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V1g))) \Rightarrow ((ap\ (ap \\
& \quad (c_2Efinite_map_2EFAPPLY\ A.27a\ A.27c)\ (ap\ (ap\ (c_2Efinite_map_2Eo_f \\
& \quad A.27a\ A.27b\ A.27c)\ V0f)\ V1g))\ V2x) = (ap\ V0f\ (ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\
& \quad A.27a\ A.27b)\ V1g)\ V2x))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\
& \quad (2^{A.27a}). (\forall V2x \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a) \\
& \quad V2x)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A.27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ A.27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A.27a)\ V2x)\ V1t))))))
\end{aligned} \tag{31}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c & \Rightarrow (\forall V0f \in (A_27b^{A_27c}). (\forall V1f1 \in (ty_2Efinite_map_2E fmap \\ & A_27a\ A_27c). (\forall V2f2 \in (ty_2Efinite_map_2E fmap\ A_27a\ A_27c). \\ & ((ap\ (ap\ (c_2Efinite_map_2Eo_f\ A_27a\ A_27c\ A_27b)\ V0f)\ (ap\ (ap \\ & (c_2Efinite_map_2EFUNION\ A_27a\ A_27c)\ V1f1)\ V2f2)) = (ap\ (ap\ (\\ & c_2Efinite_map_2EFUNION\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Efinite_map_2Eo_f \\ & A_27a\ A_27c\ A_27b)\ V0f)\ V1f1))\ (ap\ (ap\ (c_2Efinite_map_2Eo_f \\ & A_27a\ A_27c\ A_27b)\ V0f)\ V2f2)))))) \end{aligned}$$