

thm_2EfixedPoint_2Efnsum__empty
(TMNwEwqVR1sZrdLya6SbQzdBwCeqDb9hi5n)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_IN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 6 We define $c_2Ebool_2E_5C_2E_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_21 2) (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V2z \in 2.V2z)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \tag{3}$$

Definition 9 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_21 2) (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V2z \in 2.V2z)))$

Definition 10 We define $c_2EfixedPoint_2Efnsum$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f1 \in ((2^{A_27a})^{A_27b}).\lambda V1$

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 13 We define $c_2EfixedPoint_2Eempty$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0X \in A_27a.(c_2Epred_set_2EEMPTY A_27a))$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{5}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{6}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \tag{7}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow ((\forall V0s \in (2^{A_27a}).((ap (ap (c_2Epred_set_2EUNION A_27a) (c_2Epred_set_2EEMPTY A_27a)) V0s) = V0s)) \wedge (\forall V1s \in (2^{A_27a}).((ap (ap (c_2Epred_set_2EUNION A_27a) V1s) (c_2Epred_set_2EEMPTY A_27a)) = V1s))) \tag{8}$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in ((2^{A_27b})^{A_27a}).(((ap (ap (c_2EfixedPoint_2Efnsum A_27b A_27a) V0f) (c_2EfixedPoint_2Eempty A_27a A_27b)) = V0f) \wedge ((ap (ap (c_2EfixedPoint_2Efnsum A_27b A_27a) (c_2EfixedPoint_2Eempty A_27a A_27b)) V0f) = V0f)))$$