thm_2EfixedPoint_2Elfp__fnsum (TMTRvS-DSykVC9n8sj7HPPwKz8F3NpSTcKjB)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 5 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 6 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

Definition 7 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A\_$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_2}$

Definition 11 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ ($

Definition 12 *We define* c_2EfixedPoint_2Eclosed *to be* $\lambda A\_27a : \iota.\lambda V0f \in ((2^{A\_27a})^{(2^{A\_27a})}).\lambda V1X \in (2^{A\_2}$

Definition 13 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{2}$$

Definition 14 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b\in((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{3}$$

Definition 15 *We define* c_2Epred__set_2EBIGINTER *to be* $\lambda A\_27a:\iota.\lambda V0P\in(2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 16 *We define* c_2EfixedPoint_2Elfp *to be* $\lambda A\_27a:\iota.\lambda V0f\in((2^{A\_27a})^{(2^{A\_27a})}).(ap$ (c_2Epred__set

Definition 17 *We define* c_2EfixedPoint_2Emonotone *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in((2^{A\_27b})^{(2^{A\_27a})}).$

Definition 18 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t\in$

Definition 19 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ (c.$

Definition 20 *We define* c_2EfixedPoint_2Efnsum *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f1\in((2^{A\_27a})^{A\_27b}).\lambda V1$

Definition 21 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t\in 2.V0t))$.

Definition 22 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t\in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Assume the following.

$$True \tag{4}$$

Assume the following.

$$(\forall V0t1\in 2.(\forall V1t2\in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t\in 2.((\forall V1x\in A\_27a.(p\ V0t))\Leftrightarrow(p\ V0t))) \tag{6}$$

Assume the following.

$$(\forall V0t\in 2.(((p\ V0t)\Rightarrow False)\Rightarrow(\neg(p\ V0t)))) \tag{7}$$

Assume the following.

$$(\forall V0t\in 2.((\neg(p\ V0t))\Rightarrow((p\ V0t)\Rightarrow False))) \tag{8}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{9}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x))))\Leftrightarrow(\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x))\wedge(p\ (ap\ V1Q\ V2x))))\Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x)))\wedge(\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P)\wedge(\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x))))\Leftrightarrow(\forall V3x \in A\_27a.((p\ V0P)\wedge(p\ (ap\ V1Q\ V3x))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P)\vee(p\ (ap\ V1Q\ V2x))))\Leftrightarrow((p\ V0P)\vee(\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x))))))) \tag{17}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A)\vee((p\ V1B)\vee(p\ V2C)))\Leftrightarrow(((p\ V0A)\vee(p\ V1B))\vee(p\ V2C)))))) \tag{18}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A)\vee(p\ V1B))\Leftrightarrow((p\ V1B)\vee (p\ V0A))))) \tag{19}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0f \in ((2^{A\_27a})^{(2^{A\_27a})}). ((p\ (ap\ (\mathsf{c\_2EfixedPoint\_2Emonotone}\ A\_27a\ A\_27a)\ V0f))\Rightarrow((p\ (ap\ (ap\ (\mathsf{c\_2EfixedPoint\_2Eclosed}\ A\_27a)\ V0f)\ (ap\ (\mathsf{c\_2EfixedPoint\_2Elfp}\ A\_27a)\ V0f)))\wedge(\forall V1X \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2EfixedPoint\_2Eclosed}\ A\_27a)\ V0f)\ V1X))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (\mathsf{c\_2EfixedPoint\_2Elfp}\ A\_27a)\ V0f))\ V1X))))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0f \in ((2^{A\_27a})^{(2^{A\_27a})}). ((p\ (ap\ (\mathsf{c\_2EfixedPoint\_2Emonotone}\ A\_27a\ A\_27a)\ V0f))\Rightarrow(\forall V1X \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ V0f\ V1X))\ V1X))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (\mathsf{c\_2EfixedPoint\_2Elfp}\ A\_27a)\ V0f))\ V1X)))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f1 \in ((2^{A\_27b})^{(2^{A\_27a})}).(\forall V1f2 \in ((2^{A\_27b})^{(2^{A\_27a})}). (((p\ (ap\ (\mathsf{c\_2EfixedPoint\_2Emonotone}\ A\_27a\ A\_27b)\ V0f1))\wedge(p\ (ap\ (\mathsf{c\_2EfixedPoint\_2Emonotone}\ A\_27a\ A\_27b)\ V1f2)))\Rightarrow(p\ (ap\ (\mathsf{c\_2EfixedPoint\_2Emonotone}\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2EfixedPoint\_2Efnsum}\ A\_27b\ (2^{A\_27a}))\ V0f1)\ V1f2)))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0f \in ((2^{A\_27b})^{A\_27a}).(\forall V1g \in ((2^{A\_27b})^{A\_27a}). (\forall V2X \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27b)\ (ap\ V0f\ V2X))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EfixedPoint\_2Efnsum}\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2X)))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27b)\ (ap\ V1g\ V2X))\ (ap\ (ap\ (ap\ (\mathsf{c\_2EfixedPoint\_2Efnsum}\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2X))))))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).(\forall V2u \in (2^{A\_27a}).(((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V0s)\ V1t)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V1t)\ V2u))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ V0s)\ V2u)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (31)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor ((p\ V1q) \lor (p\ V2r))) \land (((p\ V0p) \lor ((\neg(p\ V2r)) \lor (\neg(p\ V1q)))) \land (((p\ V1q) \lor ((\neg(p\ V2r)) \lor (\neg(p\ V0p)))) \land ((p\ V2r) \lor ((\neg(p\ V1q)) \lor (\neg(p\ V0p))))))))))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \land (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor ((\neg(p\ V1q)) \lor (\neg(p\ V2r)))) \land (((p\ V1q) \lor (\neg(p\ V0p))) \land ((p\ V2r) \lor (\neg(p\ V0p))))))))) \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \lor (p\ V2r))) \Leftrightarrow (((p\ V0p) \lor (\neg(p\ V1q))) \land (((p\ V0p) \lor (\neg(p\ V2r))) \land ((p\ V1q) \lor ((p\ V2r) \lor (\neg(p\ V0p)))))))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg (p\ V2r))) \wedge ((\neg (p\ V1q)) \vee ((p\ V2r) \vee (\neg (p\ V0p)))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg (p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg (p\ V1q)) \vee (\neg (p\ V0p))))))) \quad (36)$$

Theorem 1

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0f1 \in ((2^{A\_27a})^{(2^{A\_27a})}). (\forall V1f2 \in ((2^{A\_27a})^{(2^{A\_27a})}).(((p\ (ap\ (\mathsf{c\_2EfixedPoint\_2Emonotone}\ A\_27a\ A\_27a)\ V0f1)) \wedge (p\ (ap\ (\mathsf{c\_2EfixedPoint\_2Emonotone}\ A\_27a\ A\_27a)\ V1f2))) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (\mathsf{c\_2EfixedPoint\_2Elfp}\ A\_27a)\ V0f1))\ (ap\ (\mathsf{c\_2EfixedPoint\_2Elfp}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2EfixedPoint\_2Efnsum}\ A\_27a\ (2^{A\_27a}))\ V0f1)\ V1f2)))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2ESUBSET}\ A\_27a)\ (ap\ (\mathsf{c\_2EfixedPoint\_2Elfp}\ A\_27a)\ V1f2))\ (ap\ (\mathsf{c\_2EfixedPoint\_2Elfp}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2EfixedPoint\_2Efnsum}\ A\_27a\ (2^{A\_27a}))\ V0f1)\ V1f2))))))))$$