

thm_2Efloat_2ECLOSEST_IS_CLOSEST (TM- MmUfC4N3DW6gS74LL9XLxZoH6orE4YQ7V)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2E_K$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.(\lambda V0x \in A.27a.(\lambda V1y \in A.27b.V0x))$

Definition 4 We define $c_2Ecombin_2E_S$ to be $\lambda A.\lambda 27a : \iota.\lambda A.\lambda 27b : \iota.\lambda A.\lambda 27c : \iota.(\lambda V0f \in ((A.27c^{A.27b})^{A.27a}))$

Definition 5 We define $c_2Ecombin_2E_I$ to be $\lambda A.\lambda 27a : \iota.(ap (ap (c_2Ecombin_2E_S A.27a (A.27a^{A.27a})) A.27a))$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap V0P (ap (c_2Emin_2E_40 A.27a) P)))$

Let $ty_2Ehreal_2E_hreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2E_hreal \tag{1}$$

Let $ty_2Epair_2E_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2E_prod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2E_real : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2E_real \tag{3}$$

Let $c_2Erealax_2E_real_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2E_real_REP_CLASS \in ((2^{(ty_2Epair_2E_prod\ ty_2Ehreal_2E_hreal\ ty_2Ehreal_2E_hreal)})\ ty_2Erealax_2E_real) \tag{4}$$

Definition 8 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap (ap (c_2Emin_2E_3D (2^{A.27a})) P)))$

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (ty$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (5)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 10 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 11 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (8)$$

Definition 12 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 13 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (9)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (11)$$

Definition 14 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (12)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (13)$$

Definition 15 We define $c_Erealax_Ereal_lt$ to be $\lambda V0t1 \in ty_Erealax_Ereal.\lambda V1t2 \in ty_Erealax_Ereal$.

Definition 16 We define c_Ebool_EF to be $(ap (c_Ebool_E21 2) (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_Emin_E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 18 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_E21 2))$.

Definition 19 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$.

Definition 20 We define $c_Ebool_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21 2) (\lambda V2t \in 2.V2t))))$.

Definition 21 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_Ebool_E21 2) (V1t1 V2t2)))))$.

Definition 22 We define c_Ereal_Eabs to be $\lambda V0x \in ty_Erealax_Ereal.(ap (ap (ap (c_Ebool_ECOND A_27a 2) (V0x V0x)) (V0x V0x)))$.

Definition 23 We define c_Ebool_EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$.

Definition 24 We define $c_Eieeee_Eis_closest$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_Erealax_Ereal^{A_27a}).\lambda V1s \in (ty_Erealax_Ereal^{A_27a})$.

Definition 25 We define $c_Eieeee_Eclosest$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_Erealax_Ereal^{A_27a}).\lambda V1p \in (ty_Erealax_Ereal^{A_27a})$.

Definition 26 We define $c_Epred_set_EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_EF)$.

Definition 27 We define $c_Ebool_E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21 2) (\lambda V2t \in 2.V2t))))$.

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EABS_prod \\ A_27a A_27b \in ((ty_Epair_Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (14)$$

Definition 28 We define c_Epair_E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_Ebool_E21 2) (V0x V1y))$.

Let $c_Epred_set_EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epred_set_EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{((ty_Epair_Eprod A_27a 2)^{A_27b})}) \end{aligned} \quad (15)$$

Definition 29 We define $c_Epred_set_EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_Ebool_E21 2) (V0x V1s))$.

Definition 30 We define $c_Epred_set_EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_Ebool_E21 2) (V0s V0s))$.

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p\ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\forall V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q)))))) \Leftrightarrow ((\forall V3x \in A_27a. (p\ (ap\ V1P\ V3x))) \vee (p\ V0Q)))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c.2Ecombin_2El\ A_27a)\ V0x) = V0x)) \quad (34)$$

Assume the following.

$$\begin{aligned}
& (\forall V0v \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \\
& \quad (\forall V1p \in (2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})) \\
& \quad (\forall V2s \in (2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})) \\
& \quad \quad (\forall V3x \in ty_2Erealax_2Ereal.((p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad \quad \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Enum_2Enum))))\ V2s)) \Rightarrow ((\neg(V2s = (c_2Epred_set_2EEMPTY\ (ty_2Epair_2Eprod \\
& \quad \quad \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))))) \Rightarrow \\
& \quad \quad \quad ((p\ (ap\ (ap\ (ap\ (ap\ (ap\ (c_2Eieeee_2Eis_closest\ (ty_2Epair_2Eprod \\
& \quad \quad \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))) \\
& \quad \quad \quad V0v)\ V2s)\ V3x)\ (ap\ (ap\ (ap\ (ap\ (c_2Eieeee_2Eclosest\ (ty_2Epair_2Eprod \\
& \quad \quad \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))) \\
& \quad \quad \quad V0v)\ V1p)\ V2s)\ V3x))) \wedge ((\exists V4b \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad \quad \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)).(p\ (ap\ (\\
& \quad \quad \quad ap\ (ap\ (ap\ (c_2Eieeee_2Eis_closest\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad \quad \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))))\ V0v)\ V2s) \\
& \quad \quad \quad V3x)\ V4b)) \wedge (p\ (ap\ V1p\ V4b))) \Rightarrow (p\ (ap\ V1p\ (ap\ (ap\ (ap\ (ap\ (c_2Eieeee_2Eclosest \\
& \quad \quad \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad \quad \quad ty_2Enum_2Enum))))\ V0v)\ V1p)\ V2s)\ V3x))))))))) \\
& \quad \quad \quad (35)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (39)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (40)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee (\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{45}$$

Theorem 1

$$\begin{aligned}
& (\forall V0v \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_ \\
& (\forall V1p \in (2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum \\
& (\forall V2x \in ty_2Erealax_2Ereal. (\forall V3s \in (2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& ((p\ (ap\ (c_2Epred_set_2EFINITE\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))\ V3s)) \Rightarrow (\\
& (\neg(V3s = (c_2Epred_set_2EEMPTY\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))))) \Rightarrow (p\ (ap \\
& (ap\ (ap\ (ap\ (c_2Eieeee_2Eis_closest\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))\ V0v)\ V3s) \\
& V2x)\ (ap\ (ap\ (ap\ (ap\ (c_2Eieeee_2Eclosest\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))\ V0v)\ V1p) \\
& V3s)\ V2x))))))
\end{aligned}$$