thm_2Efloat_2EDEFLOAT__FLOAT__ZEROSIGN__ROUND (TMGW6RR5R8rWiUepvYFnCbiPmgNdT8LU1V9)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 9 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 11 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 12 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{7}$$

Let $c\_2Eieee\_2Efracwidth : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efracwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{8}$$

Definition 13 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Let $c\_2Eieee\_2Eexpwidth : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexpwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{11}$$

Definition 14 *We define* c_2Eieee_2Eemax *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enu$

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{12}$$

Definition 16 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 17 *We define* c_2Eieee_2Ebottomfloat *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Definition 18 *We define* c_2Eieee_2Etopfloat *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 19 *We define* c_2Eieee_2Eplus__infinity *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ t$

Definition 20 *We define* c_2Eieee_2Eminus__infinity *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$

Let $ty\_2Eieee\_2Eroundmode : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eieee\_2Eroundmode \tag{13}$$

Let $c\_2Eieee\_2ETo\_\_nearest : \iota$ be given. Assume the following.

$$c\_2Eieee\_2ETo\_\_nearest \in ty\_2Eieee\_2Eroundmode \tag{14}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{15}$$

Let $c\_2Eieee\_2Eround : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eround \in ((((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))^{ty\_2Erealax\_2Ereal})^{ty\_2Eieee\_2Eroundmode})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{16}$$

Definition 21 *We define* c_2Eieee_2Eminus__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Definition 22 *We define* c_2Eieee_2Eplus__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Let $c\_2Eieee\_2Efraction : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efraction \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2} \tag{17}$$

Let $c\_2Eieee\_2Eexponent : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexponent \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2} \tag{18}$$

Definition 23 *We define* c_2Eieee_2Eis__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 24 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 25 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 26 *We define* c_2Eieee_2Ezerosign *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 27 *We define* c_2Eieee_2Efloat__format *to be* $(ap\ (ap\ ($c_2Epair_2E_2C $ty\_2Enum\_2Enum\ ty\_2E$

Let $c\_2Eieee\_2Eis\_\_valid : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eis\_\_valid \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2E \tag{19}$$

Let $ty\_2Eieee\_2Efloat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eieee\_2Efloat \tag{20}$$

Let $c\_2Eieee\_2Edefloat : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Edefloat \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))^{ty\_2Eieee\_2Efloat}) \tag{21}$$

Let $c\_2Eieee\_2Efloat : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efloat \in (ty\_2Eieee\_2Efloat^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enu \tag{22}$$

Assume the following.

$$True \tag{23}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{29}$$

Assume the following.

$$(\forall V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ ((p\ (ap\ (ap\ c\_2Eieee\_2Eis\_\_valid\ V0X)\ (ap\ \mathsf{c\_2Eieee\_2Eminus\_\_infinity}\ V0X)))\wedge((p\ (ap\ (ap\ c\_2Eieee\_2Eis\_\_valid\ V0X)\ (ap\ \mathsf{c\_2Eieee\_2Eplus\_\_infinity}\ V0X)))\wedge((p\ (ap\ (ap\ c\_2Eieee\_2Eis\_\_valid\ V0X)\ (ap\ \mathsf{c\_2Eieee\_2Etopfloat}\ V0X)))\wedge((p\ (ap\ (ap\ c\_2Eieee\_2Eis\_\_valid\ V0X)\ (ap\ \mathsf{c\_2Eieee\_2Ebottomfloat}\ V0X)))\wedge((p\ (ap\ (ap\ c\_2Eieee\_2Eis\_\_valid\ V0X)\ (ap\ \mathsf{c\_2Eieee\_2Eplus\_\_zero}\ V0X)))\wedge(p\ (ap\ (ap\ c\_2Eieee\_2Eis\_\_valid\ V0X)\ (ap\ \mathsf{c\_2Eieee\_2Eminus\_\_zero}\ V0X)))))))) \tag{30}$$

Assume the following.

$$(\forall V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum).\ (\forall V1x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ c\_2Eieee\_2Eis\_\_valid\ V0X)\ (ap\ (ap\ (ap\ c\_2Eieee\_2Eround\ V0X)\ c\_2Eieee\_2ETo\_\_nearest)\ V1x))))) \tag{31}$$

Assume the following.

$$((\forall V0a \in ty\_2Eieee\_2Efloat.((ap\ c\_2Eieee\_2Efloat\ (ap\ c\_2Eieee\_2Edefloat\ V0a)) = V0a))\wedge(\forall V1r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)).((p\ (ap\ (ap\ c\_2Eieee\_2Eis\_\_valid\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ V1r))\Leftrightarrow((ap\ c\_2Eieee\_2Edefloat\ (ap\ c\_2Eieee\_2Efloat\ V1r)) = V1r)))) \tag{32}$$

Theorem 1

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Enum\_2Enum.\ ((ap\ c\_2Eieee\_2Edefloat\ (ap\ c\_2Eieee\_2Efloat\ (ap\ (ap\ (ap\ \mathsf{c\_2Eieee\_2Ezerosign}\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ V1b)\ (ap\ (ap\ (ap\ c\_2Eieee\_2Eround\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ c\_2Eieee\_2ETo\_\_nearest)\ V0x)))) = (ap\ (ap\ (ap\ \mathsf{c\_2Eieee\_2Ezerosign}\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ V1b)\ (ap\ (ap\ (ap\ c\_2Eieee\_2Eround\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ c\_2Eieee\_2ETo\_\_nearest)\ V0x)))))$$