

thm_2Efloat_2EERROR_IS_ZERO
(TMJU8rGJ8Em335b5EbTzqCSq2KTYJJJjG1Y)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2EIN` to be $\lambda A. \lambda a : \iota. (\lambda V0x \in A. \lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x))$

Definition 3 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 4 We define `c_2Ebool_2EET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 6 We define `c_2Ebool_2E_5C_2E_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 \ A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda a. \text{nonempty } A \Rightarrow \forall A. \lambda b. \text{nonempty } A \Rightarrow \text{c_2Epair_2EABS_prod } A \ a \ A \ b \in ((\text{ty_2Epair_2Eprod } A \ a \ A \ b))^{((2^{A-27b})^{A-27a})} \tag{2}$$

Definition 8 We define `c_2Epair_2E_2C` to be $\lambda A. \lambda a : \iota. \lambda A. \lambda b : \iota. \lambda V0x \in A. \lambda V1y \in A. \lambda V2z \in A. (\text{ap } (\text{c_2Ebool_2E_5C_2E_2F } 2))$

Let `c_2Epred_set_2EGSPEC` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. \lambda a. \text{nonempty } A \Rightarrow \forall A. \lambda b. \text{nonempty } A \Rightarrow \text{c_2Epred_set_2EGSPEC } A \ a \ A \ b \in ((2^{A-27a})^{((\text{ty_2Epair_2Eprod } A \ a \ A \ b)^{A-27b})}) \tag{3}$$

Definition 9 We define `c_2Epred_set_2EINSERT` to be $\lambda A. \lambda a : \iota. \lambda V0x \in A. \lambda V1s \in (2^{A-27a}). (\text{ap } (\text{c_2Ebool_2E_5C_2E_2F } 2))$

Definition 10 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 11 We define $c_Epred_set_2EMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2EF)$.

Definition 12 We define $c_Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).(ap (c_Ebool_2E_21) 2)$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (4)$$

Let $ty_2Eieeee_2Efloat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieeee_2Efloat \quad (5)$$

Let $c_2Eieeee_2Edefloat : \iota$ be given. Assume the following.

$$c_2Eieeee_2Edefloat \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^{ty_2Eieeee_2Efloat}) \quad (6)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (8)$$

Definition 13 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 14 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (10)$$

Definition 15 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 16 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 17 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$

Definition 18 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 19 We define `c_2Eieee_2Efloat_format` to be $(ap (ap (c_2Epair_2E_2C ty_2Eenum_2Eenum ty_2Eenum_2Eenum)))$

Let `c_2Eieee_2EFraction` : ι be given. Assume the following.

$$c_2Eieee_2EFraction \in (ty_2Eenum_2Eenum^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum (ty_2Epair_2Eprod ty_2Eenum_2Eenum))}) \quad (12)$$

Let `c_2Eieee_2Eexponent` : ι be given. Assume the following.

$$c_2Eieee_2Eexponent \in (ty_2Eenum_2Eenum^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum (ty_2Epair_2Eprod ty_2Eenum_2Eenum))}) \quad (13)$$

Definition 20 We define `c_2Eieee_2Eis_zero` to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)$

Definition 21 We define `c_2Eieee_2Elszero` to be $\lambda V0a \in ty_2Eieee_2Efloat.(ap (ap c_2Eieee_2Eis_zero c_2Efloat_2E_2E))$

Definition 22 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 23 We define `c_2Eieee_2Eis_denormal` to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)$

Definition 24 We define `c_2Eieee_2Elsdenormal` to be $\lambda V0a \in ty_2Eieee_2Efloat.(ap (ap c_2Eieee_2Eis_denormal c_2Efloat_2E_2E))$

Let `c_2Eieee_2Eexpwidth` : ι be given. Assume the following.

$$c_2Eieee_2Eexpwidth \in (ty_2Eenum_2Eenum^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)}) \quad (14)$$

Let `c_2Earithmetic_2EEXP` : ι be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Eenum_2Eenum^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum}) \quad (15)$$

Let `c_2Earithmetic_2E_2D` : ι be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Eenum_2Eenum^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum}) \quad (16)$$

Definition 25 We define `c_2Eieee_2Eemax` to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)$

Definition 26 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 27 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a))))$

Definition 28 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in ty_2Eenum_2Eenum.\lambda V1n \in ty_2Eenum_2Eenum$

Definition 29 We define `c_2Eieee_2Eis_normal` to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)$

Definition 30 We define `c_2Eieee_2Elsnormal` to be $\lambda V0a \in ty_2Eieee_2Efloat.(ap (ap c_2Eieee_2Eis_normal c_2Efloat_2E_2E))$

Definition 31 We define `c_2Eieee_2Efinite` to be $\lambda V0a \in ty_2Eieee_2Efloat.(ap (ap c_2Ebool_2E_5C_2E_2F a))$

Let $c_2Eieeee_2Efloat : \iota$ be given. Assume the following.

$$c_2Eieeee_2Efloat \in (ty_2Eieeee_2Efloat^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum))}) \quad (17)$$

Let $ty_2Eieeee_2Eroundmode : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieeee_2Eroundmode \quad (18)$$

Let $c_2Eieeee_2ETO_nearest : \iota$ be given. Assume the following.

$$c_2Eieeee_2ETO_nearest \in ty_2Eieeee_2Eroundmode \quad (19)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (20)$$

Let $c_2Eieeee_2Eround : \iota$ be given. Assume the following.

$$c_2Eieeee_2Eround \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^{ty_2Erealax_2Ereal})^{ty_2Eieeee_2Eroundmode})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \quad (21)$$

Let $c_2Eieeee_2Evalof : \iota$ be given. Assume the following.

$$c_2Eieeee_2Evalof \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))})^{c_2Eieeee_2Evalof}) \quad (22)$$

Definition 32 We define $c_2Eieeee_2EVal$ to be $\lambda V0a \in ty_2Eieeee_2Efloat.(ap\ (ap\ c_2Eieeee_2Evalof\ c_2Eieeee_2Efloat)\ a)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (23)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (24)$$

Definition 33 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ty_2Erealax_2Ereal)\ a)$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (25)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)} \quad (26)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (27)$$

Definition 34 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 35 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (28)$$

Definition 36 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 37 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 38 We define $c_2Efloat_2Eerror$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ c_2Ereal_2Ereal_sub))$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (29)$$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (30)$$

Definition 39 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 40 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 41 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(A_27a\ t1\ t2)))$

Definition 42 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND))))$

Definition 43 We define $c_2Eieee_2Eis_closest$ to be $\lambda A_27a : \iota.\lambda V0v \in (ty_2Erealax_2Ereal^{A_27a}).\lambda V1s$

Let $c_2Eieee_2Eto_ninfinitiy : \iota$ be given. Assume the following.

$$c_2Eieee_2Eto_ninfinitiy \in ty_2Eieee_2Eroundmode \quad (31)$$

Let $c_2Eieee_2Eto_pinfinitiy : \iota$ be given. Assume the following.

$$c_2Eieee_2Eto_pinfinitiy \in ty_2Eieee_2Eroundmode \quad (32)$$

Let $c_2Eieee_2Efracwidth : \iota$ be given. Assume the following.

$$c_2Eieee_2Efracwidth \in (ty_2Eenum_2Eenum^{(ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)}) \quad (33)$$

Definition 44 We define $c_2Eieee_2Etopfloat$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Eenum_2Eenum\ ty_2Eenum_2Eenum)$

Definition 45 We define $c_2Ereal_2Ereal_gt$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 46 We define $c_2Eieeee_2Ebottomfloat$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$.
Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (34)$$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (35)$$

Definition 47 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. (ap $c_2Erealax_2Ereal_2EABS$)

Definition 48 We define $c_2Eieeee_2Ebias$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$.

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (36)$$

Definition 49 We define $c_2Erealax_2Ereal_2Emul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal$. $\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 50 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal$. $\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 51 We define $c_2Eieeee_2Elargest$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$.

Let $c_2Eieeee_2Efloat_2Eto_2Ezero : \iota$ be given. Assume the following.

$$c_2Eieeee_2Efloat_2Eto_2Ezero \in ty_2Eieeee_2Eroundmode \quad (37)$$

Let $c_2Eieeee_2Eis_2Evalid : \iota$ be given. Assume the following.

$$c_2Eieeee_2Eis_2Evalid \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (38)$$

Definition 52 We define $c_2Eieeee_2Eis_2Efinite$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$.

Definition 53 We define $c_2Eieeee_2Eclosest$ to be $\lambda A_27a : \iota$. $\lambda V0v \in (ty_2Erealax_2Ereal^{A_27a})$. $\lambda V1p \in (ty_2Erealax_2Ereal)$.

Definition 54 We define $c_2Eieeee_2Eplus_2Einfinity$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$.

Definition 55 We define $c_2Eieeee_2Eminus_2Einfinity$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$.

Definition 56 We define $c_2Eieeee_2Ethreshold$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$.

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (39)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (40)$$

Definition 57 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 58 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 59 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2Ebool$

Definition 60 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Definition 61 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_3E$

Definition 62 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 63 We define $c_2Ereal_2Ereal_ge$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 64 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (41)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (42)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)))))) \quad (43)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = c_2Enum_2E0) \wedge (V1n = c_2Enum_2E0)))))) \quad (44)$$

Assume the following.

$$((\forall V0n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D V0n) c_2Enum_2E0)) \Leftrightarrow (V0n = c_2Enum_2E0)) \wedge (\forall V1m \in ty_2Enum_2Enum.(\forall V2n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D V1m) (ap c_2Enum_2ESUC V2n))) \Leftrightarrow ((V1m = (ap c_2Enum_2ESUC V2n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D V1m) V2n)))))))))) \quad (45)$$

Assume the following.

$$True \quad (46)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (54)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (55)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ & (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ & p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\ & V5y_27)))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1)) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned}
& (\forall V0v \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \\
& \quad (\forall V1p \in (2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \\
& (\forall V2x \in ty_2Erealax_2Ereal. (\forall V3s \in (2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum} \\
& \quad ((p (ap (c_2Epred_set_2EFINITE (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))) V3s)) \Rightarrow (\\
& \quad (\neg (V3s = (c_2Epred_set_2EEMPTY (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))))) \Rightarrow (p (ap \\
& \quad (ap (ap (ap (c_2Eieeee_2Eis_closest (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))) V0v) V3s) \\
& \quad V2x) (ap (ap (ap (ap (c_2Eieeee_2Eclosest (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))) V0v) V1p) \\
& \quad V3s) V2x)))))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\
& \quad (p (ap (c_2Epred_set_2EFINITE (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))) (ap (c_2Epred_set_2EGSPEC \\
& (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& ty_2Enum_2Enum)) (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& ty_2Enum_2Enum\ ty_2Enum_2Enum))) (\lambda V1a \in (ty_2Epair_2Eprod \\
& ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& (ap (ap (c_2Epair_2E_2C (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ 2) V1a) (ap (ap c_2Eieeee_2Eis_finite \\
& ty_2Enum_2Enum\ ty_2Enum_2Enum\ 2) V1a) (ap (ap c_2Eieeee_2Eis_finite \\
& V0X) V1a))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum). \\
& \quad (\neg ((ap (c_2Epred_set_2EGSPEC (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)) (ty_2Epair_2Eprod \\
& ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))) \\
& \quad (\lambda V1a \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& ty_2Enum_2Enum\ ty_2Enum_2Enum)). (ap (ap (c_2Epair_2E_2C (ty_2Epair_2Eprod \\
& ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)) \\
& 2) V1a) (ap (ap c_2Eieeee_2Eis_finite V0X) V1a)))) = (c_2Epred_set_2EEMPTY \\
& (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& ty_2Enum_2Enum))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Eieeee_2Edefloat (ap \\
& c_2Eieeee_2Efloat (ap (ap (ap c_2Eieeee_2Eround c_2Eieeee_2Efloat_format \\
& c_2Eieeee_2ETo_nearest) V0x))) = (ap (ap (ap c_2Eieeee_2Eround \\
& c_2Eieeee_2Efloat_format) c_2Eieeee_2ETo_nearest) V0x)))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
 (\forall V0a \in ty_2Eieee_2Efloat. ((p (ap c_2Eieee_2EFinite V0a)) \Leftrightarrow \\
 (p (ap (ap c_2Eieee_2Eis_finite c_2Eieee_2Efloat_format) (\\
 ap c_2Eieee_2Edefloat V0a)))))) \quad (69)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
 (\forall V0a \in ty_2Eieee_2Efloat. ((p (ap c_2Eieee_2EFinite V0a)) \Rightarrow \\
 (p (ap (ap c_2Erealx_2Ereal_lt (ap c_2Ereal_2Eabs (ap c_2Eieee_2EVal \\
 V0a)) (ap c_2Eieee_2Ethreshold c_2Eieee_2Efloat_format)))))) \quad (70)
 \end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& \quad (\forall V1x \in ty_2Erealx_2Ereal.((ap\ (ap\ (ap\ (c_2Eieeee_2Eround \\
& \quad V0X)\ c_2Eieeee_2ETo_nearest)\ V1x) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum))))\ (ap\ (ap\ c_2Ereal_2Ereal_lte\ V1x)\ (ap\ c_2Erealx_2Ereal_neg \\
& \quad (ap\ c_2Eieeee_2Ethreshold\ V0X))))\ (ap\ c_2Eieeee_2Eminus_infinity \\
& \quad V0X))\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))))\ (ap\ (ap\ c_2Ereal_2Ereal_ge \\
& \quad V1x)\ (ap\ c_2Eieeee_2Ethreshold\ V0X))))\ (ap\ c_2Eieeee_2Eplus_infinity \\
& \quad V0X))\ (ap\ (ap\ (ap\ (ap\ (c_2Eieeee_2Eclosest\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))))\ (ap\ c_2Eieeee_2Evalof \\
& \quad V0X))\ (\lambda V2a \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ ty_2Enum_2Enum)).(ap\ c_2Earithmetric_2EEVEN \\
& \quad (ap\ c_2Eieeee_2Efraction\ V2a))))\ (ap\ (c_2Epred_set_2EGSPEC\ (\\
& \quad ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)))\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ ty_2Enum_2Enum)))\ (\lambda V3a \in (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ ty_2Enum_2Enum))\ 2)\ V3a)\ (ap\ (ap\ c_2Eieeee_2Eis_finite \\
& \quad V0X)\ V3a))))\ V1x))))))\ (\forall V4X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)).(\forall V5x \in ty_2Erealx_2Ereal.((ap\ (ap\ (\\
& \quad ap\ c_2Eieeee_2Eround\ V4X)\ c_2Eieeee_2Efloat_To_zero)\ V5x) = (\\
& \quad ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))\ (ap\ (ap\ c_2Erealx_2Ereal_lt \\
& \quad V5x)\ (ap\ c_2Erealx_2Ereal_neg\ (ap\ c_2Eieeee_2Elargest\ V4X))))\ \\
& \quad (ap\ c_2Eieeee_2Ebottomfloat\ V4X))\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum))))\ (ap\ (ap\ c_2Ereal_2Ereal_gt\ V5x)\ (ap\ c_2Eieeee_2Elargest \\
& \quad V4X))))\ (ap\ c_2Eieeee_2Etopfloat\ V4X))\ (ap\ (ap\ (ap\ (ap\ (c_2Eieeee_2Eclosest \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum))))\ (ap\ c_2Eieeee_2Evalof\ V4X))\ (\lambda V6x \in (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& \quad c_2Ebool_2ET))\ (ap\ (c_2Epred_set_2EGSPEC\ (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)) \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)))\ (\lambda V7a \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)).(ap\ (ap\ (\\
& \quad c_2Epair_2E_2C\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ ty_2Enum_2Enum))\ 2)\ V7a)\ (ap\ (ap\ c_2Ebool_2E_2F_5C \\
& \quad (ap\ (ap\ c_2Eieeee_2Eis_finite\ V4X)\ V7a))\ (ap\ (ap\ c_2Ereal_2Ereal_lte \\
& \quad (ap\ c_2Ereal_2Eabs\ (ap\ (ap\ c_2Eieeee_2Evalof\ V4X)\ V7a))))\ (ap\ c_2Ereal_2Eabs \\
& \quad V5x))))))\ V5x))))))\ (\forall V8X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)).(\forall V9x \in ty_2Erealx_2Ereal.((ap\ (ap\ (\\
& \quad ap\ c_2Eieeee_2Eround\ V8X)\ c_2Eieeee_2ETo_pinfinity)\ V9x) = (ap \\
& \quad (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ ty_2Enum_2Enum)))\ (ap\ (ap\ c_2Erealx_2Ereal_lt \\
& \quad V9x)\ (ap\ c_2Erealx_2Ereal_neg\ (ap\ c_2Eieeee_2Elargest\ V8X))))\ \\
& \quad (ap\ c_2Eieeee_2Ebottomfloat\ V8X))\ (ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum))))\ (ap\ (ap\ c_2Ereal_2Ereal_gt\ V9x)\ (ap\ c_2Eieeee_2Elargest \\
& \quad V8X))))\ (ap\ c_2Eieeee_2Eplus_infinity\ V8X))\ (ap\ (ap\ (ap\ (ap\ (c_2Eieeee_2Eclosest \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum))))\ (ap\ c_2Eieeee_2Evalof\ V8X))\ (\lambda V10x \in (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)). \\
& \quad c_2Ebool_2ET))\ (ap\ (c_2Epred_set_2EGSPEC\ (ty_2Epair_2Eprod \\
& \quad ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)) \\
& \quad (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)))\ (\lambda V11a \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)))\ (\lambda V11a \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO)))) \wedge ((\forall V17n \in ty_2Enum_2Enum.(\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = V0n) \wedge (((ap\ c_2Enumeral_2EiZ\ (\\
& ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (\\
& ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (\\
& ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ c_2Earithmetic_2EZERO)) = (ap\ c_2Enum_2ESUC\ V0n)) \wedge (((ap \\
& c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = (ap\ c_2Earithmetic_2EBIT2 \\
& (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (\\
& ((ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT2 \\
& V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = (ap\ c_2Earithmetic_2EBIT1 \\
& (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Earithmetic_2EZERO) \\
& V0n)) = (ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC \\
& (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ c_2Earithmetic_2EZERO)) = (\\
& ap\ c_2Enumeral_2EiiSUC\ V0n)) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (\\
& ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT1 \\
& V1m))) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT1\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT1\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT1\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& (ap\ c_2Earithmetic_2EBIT2\ V0n))\ (ap\ c_2Earithmetic_2EBIT2\ V1m))) = \\
& (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiiSUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT1\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Enumeral_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Earithmetic_2EBIT2\ V0n)) = (ap\ c_2Earithmetic_2EBIT2\ (ap\ c_2Earithmetic_2EBIT1\ V0n))) \wedge ((ap\ c_2Enumeral_2EiDUB\ c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO)))) \\
& \hspace{15em} (74)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((ap\ (ap\ c_2Earithmetic_2E_2A\ c_2Earithmetic_2EZERO)\ V0n) = c_2Earithmetic_2EZERO) \wedge \\
& \quad (((ap\ (ap\ c_2Earithmetic_2E_2A\ V0n)\ c_2Earithmetic_2EZERO) = \\
& \quad c_2Earithmetic_2EZERO) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2EBIT1\ V0n))\ V1m) = (ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ c_2Enumeral_2EiDUB\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0n)\ V1m)))\ V1m))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ c_2Earithmetic_2EBIT2\ V0n))\ V1m) = (ap\ c_2Enumeral_2EiDUB\ (ap\ c_2Enumeral_2EiZ\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0n)\ V1m))\ V1m)))))))))) \\
& \hspace{15em} (75)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\
& \hspace{15em} (76)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\
& \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC\ A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \\
& \hspace{15em} (77)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y) = (ap\ (ap\ c_2Erealax_2Ereal_add\ V1y)\ V0x)))) \\
& \hspace{15em} (78)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad (\forall V2z \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V1y)\ V2z)) = (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y))\ V2z)))))) \\
& \hspace{15em} (79)
\end{aligned}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = V0x)) \quad (80)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (81)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z)))))) \quad (82)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(\forall V2z \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z)))))) \quad (83)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x)) \quad (84)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)))))) \quad (85)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x)) \quad (86)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (87)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (88)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \quad (89)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) (ap (ap c_2Erealax_2Ereal_add V0x) V2z)))) \Leftrightarrow (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z)))))) \quad (90)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \vee (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))))) \quad (91)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z)))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z)))))) \quad (92)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V1y) V2z)))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt V0x) V2z)))))) \quad (93)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z)))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V2z)))))) \quad (94)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (95)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V1y))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_mul V0x) V1y))))))
\end{aligned} \tag{96}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Ereal_2Ereal_sub V0x) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Erealax_2Ereal_add (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y)) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V1y))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V1y))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0y) V1x))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V2z)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap (ap c_2Erealax_2Ereal_add V0x) V2z))))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x))))))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg \\
& (ap c_2Erealax_2Ereal_neg V0x)) = V0x))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n))))))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num \\
& V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num \\
& (ap (ap c_2Earithmetic_2E_2A V0m) V1n))))))
\end{aligned} \tag{109}$$

Assume the following.

$$\begin{aligned} ((ap\ c_2Ereal_2Eabs\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = \\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) \end{aligned} \quad (110)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (111)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (112)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (113)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (114)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False) \Rightarrow False)) \quad (115)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (116)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (117)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (118)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\\ \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (119)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (120)$$

Theorem 1

$$(\forall V0a \in ty_2Eieee_2Efloat.(\forall V1x \in ty_2Erealax_2Ereal.(((p (ap c_2Eieee_2EFinite V0a)) \wedge ((ap c_2Eieee_2EVal V0a) = V1x)) \Rightarrow ((ap c_2Efloat_2EError V1x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))$$