

thm_2Efloat_2EFLOAT__ADD__FINITE (TMT39MrdEozXR7tjmuXETiiPXkYE5tzGrH)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x)$.

Let $ty_2Eieeee_2ERoundmode : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieeee_2ERoundmode \tag{1}$$

Let $c_2Eieeee_2ETo_nearest : \iota$ be given. Assume the following.

$$c_2Eieeee_2ETo_nearest \in ty_2Eieeee_2ERoundmode \tag{2}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{5}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{6}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{7}$$

Definition 5 We define $c_Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_Emin_2E_3D (2^{A_27a})))$

Definition 6 We define c_Eenum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_Eenum_2EABS_num ($

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

Definition 8 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_Ebool_2E_21 2) (\lambda V2t \in 2. ($

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (9)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2E_2C$

Definition 13 We define $c_2Eieee_2Efloat_format$ to be $(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \quad (11)$$

Let $c_2Eieee_2Eround : \iota$ be given. Assume the following.

$$c_2Eieee_2Eround \in (((ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum))^{ty_2Erealax_2Ereal})^{ty_2Eieee_2Eroundmode})^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (12)$$

Let $ty_2Eieee_2Efloat : \iota$ be given. Assume the following.

$$nonempty ty_2Eieee_2Efloat \quad (13)$$

Let $c_2Eieee_2Efloat : \iota$ be given. Assume the following.

$$c_2Eieee_2Efloat \in (ty_2Eieee_2Efloat)^{(ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum ty_2Enum_2Enum))^{ty_2Erealax_2Ereal}} \quad (14)$$

Let $c_Eieee_Edefloat : \iota$ be given. Assume the following.

$$c_Eieee_Edefloat \in ((ty_Epair_Eprod\ ty_Eenum_Eenum\ (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum))^{ty_Eieee_Efloat}) \quad (15)$$

Let $c_Eieee_Eevalof : \iota$ be given. Assume the following.

$$c_Eieee_Eevalof \in ((ty_Erealax_Ereal)^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum))}) \quad (16)$$

Definition 14 We define c_Eieee_Eeval to be $\lambda V0a \in ty_Eieee_Efloat.(ap\ (ap\ c_Eieee_Eevalof\ c_Eenum_Eenum\ a))$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \quad (17)$$

Let $c_Erealax_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Erealax_Ereal}) \quad (18)$$

Definition 15 We define c_Eemin_E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 16 We define $c_Erealax_Ereal_REP$ to be $\lambda V0a \in ty_Erealax_Ereal.(ap\ (c_Eemin_E40\ (c_Eenum_Eenum\ a)))$

Let $c_Erealax_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_neg \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (19)$$

Let $c_Erealax_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_eq \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (20)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal)^{(2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})} \quad (21)$$

Definition 17 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal).$

Definition 18 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal_neg)$

Let $c_Erealax_Etrealm_add : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_add \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (22)$$

Definition 19 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Definition 20 We define $c_Ereal_Ereal_sub$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Definition 21 We define c_Efloat_Eerror to be $\lambda V0x \in ty_Erealax_Ereal.(ap (ap c_Ereal_Ereal_sub$

Let $c_Eieeee_ETo_ninfinitiy : \iota$ be given. Assume the following.

$$c_Eieeee_ETo_ninfinitiy \in ty_Eieeee_ERoundmode \quad (23)$$

Definition 22 We define c_Ebool_Eef to be $(ap (c_Ebool_E.21 2) (\lambda V0t \in 2.V0t))$.

Definition 23 We define c_Ebool_ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.$

Let $c_Eieeee_Esign : \iota$ be given. Assume the following.

$$c_Eieeee_Esign \in (ty_Eenum_Eenum^{(ty_Epair_Eprod ty_Eenum_Eenum (ty_Epair_Eprod ty_Eenum_Eenum} \quad (24)$$

Let $c_Eieeee_EFraction : \iota$ be given. Assume the following.

$$c_Eieeee_EFraction \in (ty_Eenum_Eenum^{(ty_Epair_Eprod ty_Eenum_Eenum (ty_Epair_Eprod ty_Eenum_Eenum} \quad (25)$$

Let $c_Eieeee_Eexponent : \iota$ be given. Assume the following.

$$c_Eieeee_Eexponent \in (ty_Eenum_Eenum^{(ty_Epair_Eprod ty_Eenum_Eenum (ty_Epair_Eprod ty_Eenum_Eenum} \quad (26)$$

Definition 24 We define $c_Eieeee_Eis_zero$ to be $\lambda V0X \in (ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum$

Definition 25 We define $c_Eieeee_Eminus_zero$ to be $\lambda V0X \in (ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum$

Definition 26 We define $c_Eieeee_Eplus_zero$ to be $\lambda V0X \in (ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum$

Definition 27 We define $c_Eieeee_EZerosign$ to be $\lambda V0X \in (ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum$

Let $c_Eieeee_Eexpwidth : \iota$ be given. Assume the following.

$$c_Eieeee_Eexpwidth \in (ty_Eenum_Eenum^{(ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)} \quad (27)$$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (28)$$

Let $c_Earithmetic_E2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E2D \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (29)$$

Definition 28 We define c_Eieeee_Eemax to be $\lambda V0X \in (ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum$

Definition 29 We define $c_Eieeee_Eis_infinity$ to be $\lambda V0X \in (ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum$

Definition 30 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Eemin_E3D_3D_3E V0t) c_Ebool_E7E$

Definition 31 We define $c_EIEEE_Eis_nan$ to be $\lambda V0X \in (ty_Epair_Eprod\ ty_EEnum_EEnum\ ty_EEnum)$

Definition 32 We define $c_EIEEE_ESome_nan$ to be $\lambda V0X \in (ty_Epair_Eprod\ ty_EEnum_EEnum\ ty_EEnum)$

Definition 33 We define $c_Ebool_E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E21\ 2)\ (\lambda V2t \in 2.))$

Definition 34 We define c_EIEEE_EFadd to be $\lambda V0X \in (ty_Epair_Eprod\ ty_EEnum_EEnum\ ty_EEnum)$

Definition 35 We define $c_EIEEE_Efloat_add$ to be $\lambda V0a \in ty_EIEEE_Efloat.\lambda V1b \in ty_EIEEE_Efloat$

Let $c_EIEEE_EFfracwidth : \iota$ be given. Assume the following.

$$c_EIEEE_EFfracwidth \in (ty_EEnum_EEnum^{(ty_Epair_Eprod\ ty_EEnum_EEnum\ ty_EEnum_EEnum)}) \quad (30)$$

Let $c_EReal_EReal_of_num : \iota$ be given. Assume the following.

$$c_EReal_EReal_of_num \in (ty_ERealax_EReal^{ty_EEnum_EEnum}) \quad (31)$$

Let $c_EReal_Epow : \iota$ be given. Assume the following.

$$c_EReal_Epow \in ((ty_ERealax_EReal^{ty_EEnum_EEnum})^{ty_ERealax_EReal}) \quad (32)$$

Let $c_ERealax_ETreal_inv : \iota$ be given. Assume the following.

$$c_ERealax_ETreal_inv \in ((ty_Epair_Eprod\ ty_EHreal_EHreal\ ty_EHreal_EHreal)^{(ty_Epair_Eprod\ ty_EHreal_EHreal\ ty_EHreal_EHreal)}) \quad (33)$$

Definition 36 We define $c_ERealax_Einv$ to be $\lambda V0T1 \in ty_ERealax_EReal.(ap\ c_ERealax_EReal_ABS)$

Definition 37 We define c_EIEEE_EBias to be $\lambda V0X \in (ty_Epair_Eprod\ ty_EEnum_EEnum\ ty_EEnum)$

Let $c_ERealax_ETreal_mul : \iota$ be given. Assume the following.

$$c_ERealax_ETreal_mul \in (((ty_Epair_Eprod\ ty_EHreal_EHreal\ ty_EHreal_EHreal)^{(ty_Epair_Eprod\ ty_EHreal_EHreal\ ty_EHreal_EHreal)})^{(ty_Epair_Eprod\ ty_EHreal_EHreal\ ty_EHreal_EHreal)}) \quad (34)$$

Definition 38 We define $c_ERealax_EReal_mul$ to be $\lambda V0T1 \in ty_ERealax_EReal.\lambda V1T2 \in ty_ERealax_EReal$

Definition 39 We define c_EReal_E2F to be $\lambda V0x \in ty_ERealax_EReal.\lambda V1y \in ty_ERealax_EReal$

Definition 40 We define $c_EIEEE_Ethreshold$ to be $\lambda V0X \in (ty_Epair_Eprod\ ty_EEnum_EEnum\ ty_EEnum)$

Let $c_ERealax_ETreal_lt : \iota$ be given. Assume the following.

$$c_ERealax_ETreal_lt \in ((2^{(ty_Epair_Eprod\ ty_EHreal_EHreal\ ty_EHreal_EHreal)})^{(ty_Epair_Eprod\ ty_EHreal_EHreal\ ty_EHreal_EHreal)}) \quad (35)$$

Definition 41 We define $c_ERealax_EReal_lt$ to be $\lambda V0T1 \in ty_ERealax_EReal.\lambda V1T2 \in ty_ERealax_EReal$

Definition 42 We define `c2Ereal_2Ereal_lte` to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 43 We define `c2Ereal_2Eabs` to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECON$

Definition 44 We define `c2Eieeee_2Elszero` to be $\lambda V0a \in ty_2Eieeee_2Efloat.(ap (ap c_2Eieeee_2Eis_zero c_$

Definition 45 We define `c2Eieeee_2Eis_denormal` to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty$

Definition 46 We define `c2Eieeee_2Elsdenormal` to be $\lambda V0a \in ty_2Eieeee_2Efloat.(ap (ap c_2Eieeee_2Eis_de$

Definition 47 We define `c2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E40$

Definition 48 We define `c2Eprim_rec_2E_3C` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 49 We define `c2Eieeee_2Eis_normal` to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2$

Definition 50 We define `c2Eieeee_2Elsnormal` to be $\lambda V0a \in ty_2Eieeee_2Efloat.(ap (ap c_2Eieeee_2Eis_norm$

Definition 51 We define `c2Eieeee_2EFinite` to be $\lambda V0a \in ty_2Eieeee_2Efloat.(ap (ap c_2Ebool_2E_5C_2F (a$

Assume the following.

$$True \tag{36}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty_2Eieeee_2Efloat.(\forall V1b \in ty_2Eieeee_2Efloat. \\ & (((p (ap c_2Eieeee_2EFinite V0a)) \wedge ((p (ap c_2Eieeee_2EFinite V1b)) \wedge \\ & (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs (ap (ap c_2Erealax_2Ereal_add \\ & (ap c_2Eieeee_2EVal V0a)) (ap c_2Eieeee_2EVal V1b)))) (ap c_2Eieeee_2Ethreshold \\ & c_2Eieeee_2Efloat_format)))))) \Rightarrow ((p (ap c_2Eieeee_2EFinite (ap \\ & (ap c_2Eieeee_2Efloat_add V0a) V1b))) \wedge ((ap c_2Eieeee_2EVal (ap \\ & (ap c_2Eieeee_2Efloat_add V0a) V1b)) = (ap (ap c_2Erealax_2Ereal_add \\ & (ap (ap c_2Erealax_2Ereal_add (ap c_2Eieeee_2EVal V0a)) (ap c_2Eieeee_2EVal \\ & V1b))) (ap c_2Efloat_2Eerror (ap (ap c_2Erealax_2Ereal_add (\\ & ap c_2Eieeee_2EVal V0a)) (ap c_2Eieeee_2EVal V1b)))))))))) \end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{40}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (47)$$

Theorem 1

$$(\forall V0b \in \text{ty_2Eieee_2Efloat}. (\forall V1a \in \text{ty_2Eieee_2Efloat}. (((p \text{ (ap c_2Eieee_2EFinite V1a)}) \wedge ((p \text{ (ap c_2Eieee_2EFinite V0b)}) \wedge (p \text{ (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs (ap (ap c_2Erealax_2Ereal_add (ap c_2Eieee_2EVal V1a)) (ap c_2Eieee_2EVal V0b)))) (ap c_2Eieee_2Ethreshold c_2Eieee_2Efloat_format)))))) \Rightarrow (p \text{ (ap c_2Eieee_2EFinite (ap (ap c_2Eieee_2Efloat_add V1a) V0b)))))))$$