

thm\_2Efloat\_2EFLOAT\_\_CASES\_\_FINITE  
(TMSS-  
BiGY65HC5BhSd81vLGY2sak5qWMkFz5)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Eieeee\_2Efloat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eieeee\_2Efloat \tag{3}$$

Let  $c\_2Eieeee\_2Edefloat : \iota$  be given. Assume the following.

$$c\_2Eieeee\_2Edefloat \in ((ty\_2Epair\_2Eprod\ ty\_2Eenum\_2Eenum\ (ty\_2Epair\_2Eprod\ ty\_2Eenum\_2Eenum\ ty\_2Eenum\_2Eenum))^{ty\_2Eieeee\_2Efloat}) \tag{4}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{6}$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{8}$$

**Definition 10** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

**Definition 11** We define  $c\_2Earithmic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmic$

**Definition 12** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmic$

**Definition 13** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \tag{10}$$

**Definition 14** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

**Definition 15** We define  $c\_2Eieee\_2Efloat\_format$  to be  $(ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2E$

Let  $c\_2Eieee\_2Efraction : \iota$  be given. Assume the following.

$$c\_2Eieee\_2Efraction \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum))}) \tag{11}$$

Let  $c\_2Eieee\_2Eexpwidth : \iota$  be given. Assume the following.

$$c\_2Eieee\_2Eexpwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{12}$$

Let  $c\_Earithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEXP \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (13)$$

Let  $c\_Earithmetic\_E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2D \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (14)$$

**Definition 16** We define  $c\_EIEEE\_Eemax$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)$

Let  $c\_EIEEE\_Eexponent : \iota$  be given. Assume the following.

$$c\_EIEEE\_Eexponent \in (ty\_Enum\_Enum^{(ty\_Epair\_Eprod\ ty\_Enum\_Enum\ (ty\_Epair\_Eprod\ ty\_Enum\_Enum))}) \quad (15)$$

**Definition 17** We define  $c\_EIEEE\_Eis\_infinity$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)$

**Definition 18** We define  $c\_EIEEE\_EInfinity$  to be  $\lambda V0a \in ty\_EIEEE\_Efloat.(ap\ (ap\ c\_EIEEE\_Eis\_infinity\ c\_EIEEE\_Efloat))$

**Definition 19** We define  $c\_EIEEE\_Eis\_nan$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)$

**Definition 20** We define  $c\_EIEEE\_EIsnan$  to be  $\lambda V0a \in ty\_EIEEE\_Efloat.(ap\ (ap\ c\_EIEEE\_Eis\_nan\ c\_EIEEE\_Efloat))$

**Definition 21** We define  $c\_EIEEE\_Eis\_zero$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)$

**Definition 22** We define  $c\_EIEEE\_EIszero$  to be  $\lambda V0a \in ty\_EIEEE\_Efloat.(ap\ (ap\ c\_EIEEE\_Eis\_zero\ c\_EIEEE\_Efloat))$

**Definition 23** We define  $c\_EIEEE\_Eis\_denormal$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)$

**Definition 24** We define  $c\_EIEEE\_EIsdenormal$  to be  $\lambda V0a \in ty\_EIEEE\_Efloat.(ap\ (ap\ c\_EIEEE\_Eis\_denormal\ c\_EIEEE\_Efloat))$

**Definition 25** We define  $c\_Emin\_E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge P\ x)\ \text{of type } \iota \Rightarrow \iota).$

**Definition 26** We define  $c\_Ebool\_E\_3F$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_Emin\_E\_40\ 27a)\ P)))$

**Definition 27** We define  $c\_Eprim\_rec\_E\_3C$  to be  $\lambda V0m \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum$

**Definition 28** We define  $c\_EIEEE\_Eis\_normal$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)$

**Definition 29** We define  $c\_EIEEE\_EIsnormal$  to be  $\lambda V0a \in ty\_EIEEE\_Efloat.(ap\ (ap\ c\_EIEEE\_Eis\_normal\ c\_EIEEE\_Efloat))$

**Definition 30** We define  $c\_Ebool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_E\_21\ 2)\ (\lambda V2t \in 2.(ap\ (c\_Ebool\_E\_21\ 2)\ t2))))$

**Definition 31** We define  $c\_EIEEE\_EFinite$  to be  $\lambda V0a \in ty\_EIEEE\_Efloat.(ap\ (ap\ c\_Ebool\_E\_5C\_2F\ a))$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\forall A_{27a}. nonempty\ A_{27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{27a}. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \quad (18) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Eieee\_2Efloat. ((p\ (ap\ c\_2Eieee\_2Elsnan\ V0a)) \vee \\ & ((p\ (ap\ c\_2Eieee\_2EInfinity\ V0a)) \vee ((p\ (ap\ c\_2Eieee\_2Elnormal \\ & V0a)) \vee ((p\ (ap\ c\_2Eieee\_2Elsdenormal\ V0a)) \vee (p\ (ap\ c\_2Eieee\_2Elszero \\ & V0a)))))) \quad (19) \end{aligned}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0a \in ty\_2Eieee\_2Efloat. ((p\ (ap\ c\_2Eieee\_2Elsnan\ V0a)) \vee \\ & ((p\ (ap\ c\_2Eieee\_2EInfinity\ V0a)) \vee (p\ (ap\ c\_2Eieee\_2EFinite\ V0a)))) \end{aligned}$$