

thm\_2Efloat\_2EFLOAT\_DISTINCTFINITE  
 (TMagicMTVqbAd8usGdnizFkMxDYo8ta64y)

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**Definition 1** We define  $c_2Emin_2E_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c_2Ebool_2ET$  to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c_2Emin_2E_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x) \text{ else } \perp)$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define  $c_2Ebool_2E_3F$  to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40 A_27a) (c_2Emin_2E_3D (2^{A_27a})))) \Rightarrow \perp)$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \quad (1)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (ty\_2Epair\_2Eprod \\ & \quad A0 A1) \end{aligned} \quad (2)$$

Let  $ty\_2Eieee\_2Efloat : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Eieee\_2Efloat \quad (3)$$

Let  $c\_2Eieee\_2Edfloat : \iota$  be given. Assume the following.

$$c\_2Eieee\_2Edfloat \in ((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum))^{ty\_2Eieee\_2Efloat}) \quad (4)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (5)$$

Let  $c\_2Enum\_2EAABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EAABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (6)$$

**Definition 5** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 6** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

**Definition 7** We define  $c\_2Ebool\_2E_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E_3D\ (2^{A\_27a})))$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

**Definition 9** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 10** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 11** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 12** We define  $c\_2Emin\_2E_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o$  ( $P \Rightarrow p\ Q$ ) of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E_21\ 2)\ (\lambda V2t \in$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (10)$$

**Definition 14** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2E\_2C\ (V0x\ V1y)))$

**Definition 15** We define  $c\_2Eieee\_2Efloat\_format$  to be  $(ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)))$

Let  $c\_2Eieee\_2Efraction : \iota$  be given. Assume the following.

$$c\_2Eieee\_2Efraction \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))}) \quad (11)$$

Let  $c\_2Eieee\_2Eexpwidth : \iota$  be given. Assume the following.

$$c\_2Eieee\_2Eexpwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))}) \quad (12)$$

Let  $c\_2Earithmetic\_2EXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (13)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (14)$$

**Definition 16** We define  $c\_2Eieee\_2Eemax$  to be  $\lambda V0X \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum ty\_2Enum)$

Let  $c\_2Eieee\_2Eexponent : \iota$  be given. Assume the following.

$$c\_2Eieee\_2Eexponent \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum) (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)} \quad (15)$$

**Definition 17** We define  $c\_2Eieee\_2Eis\_infinity$  to be  $\lambda V0X \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum ty\_2Enum)$

**Definition 18** We define  $c\_2Eieee\_2Einfinity$  to be  $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap (ap c\_2Eieee\_2Eis\_infinity$

**Definition 19** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 20** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E$

**Definition 21** We define  $c\_2Eieee\_2Eis\_nan$  to be  $\lambda V0X \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum ty\_2Enum)$

**Definition 22** We define  $c\_2Eieee\_2Eisnan$  to be  $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap (ap c\_2Eieee\_2Eis\_nan c\_2Eieee\_2Eis\_infinity$

**Definition 23** We define  $c\_2Eieee\_2Eis\_zero$  to be  $\lambda V0X \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum ty\_2Enum)$

**Definition 24** We define  $c\_2Eieee\_2Elszero$  to be  $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap (ap c\_2Eieee\_2Eis\_zero c\_2Eieee\_2Eis\_infinity$

**Definition 25** We define  $c\_2Eieee\_2Eis\_denormal$  to be  $\lambda V0X \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum ty\_2Enum)$

**Definition 26** We define  $c\_2Eieee\_2Elsdenormal$  to be  $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap (ap c\_2Eieee\_2Eis\_denormal c\_2Eieee\_2Eis\_infinity$

**Definition 27** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum ty\_2Enum$

**Definition 28** We define  $c\_2Eieee\_2Eis\_normal$  to be  $\lambda V0X \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum ty\_2Enum)$

**Definition 29** We define  $c\_2Eieee\_2Elsnormal$  to be  $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap (ap c\_2Eieee\_2Eis\_normal c\_2Eieee\_2Eis\_infinity$

**Definition 30** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t))$

**Definition 31** We define  $c\_2Eieee\_2EFinite$  to be  $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap (ap c\_2Ebool\_2E\_5C\_2F (a$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (21)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (22)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (24)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). ((\neg(\forall V1x \in A\_27a. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a. (\neg(p (ap V0P V2x))))))) \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a. (p (ap V0P V3x))) \wedge (\forall V4x \in A\_27a. (p (ap V1Q V4x)))))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\exists V2x \in A_27a.(p(ap V0P V2x)) \vee (p(ap V1Q V2x)))) \Leftrightarrow \\ ((\exists V3x \in A_27a.(p(ap V0P V3x))) \vee (\exists V4x \in A_27a.(p(ap V1Q V4x))))))) \end{aligned} \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B)) \vee (p V2C)))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(p V0A) \wedge (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))) \wedge ((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (p V1B))))))) \quad (30)$$

Assume the following.

$$\begin{aligned} (\forall V0a \in ty\_2Eieee\_2Efloat.((\neg((p(ap c\_2Eieee\_2Eisnan V0a)) \wedge (p(ap c\_2Eieee\_2Einfinity V0a)))) \wedge ((\neg((p(ap c\_2Eieee\_2Eisnan V0a)) \wedge (p(ap c\_2Eieee\_2Elsnormal V0a)))) \wedge ((\neg((p(ap c\_2Eieee\_2Eisnan V0a)) \wedge (p(ap c\_2Eieee\_2Elsdenormal V0a)))) \wedge ((\neg((p(ap c\_2Eieee\_2Eisnan V0a)) \wedge (p(ap c\_2Eieee\_2Elszero V0a)))) \wedge ((\neg((p(ap c\_2Eieee\_2Einfinity V0a)) \wedge (p(ap c\_2Eieee\_2Elsnormal V0a)))) \wedge ((\neg((p(ap c\_2Eieee\_2Einfinity V0a)) \wedge (p(ap c\_2Eieee\_2Elsdenormal V0a)))) \wedge ((\neg((p(ap c\_2Eieee\_2Elsnormal V0a)) \wedge (p(ap c\_2Eieee\_2Elsdenormal V0a)))) \wedge ((\neg((p(ap c\_2Eieee\_2Elsnormal V0a)) \wedge (p(ap c\_2Eieee\_2Elszero V0a)))) \wedge ((\neg((p(ap c\_2Eieee\_2Elsdenormal V0a)) \wedge (p(ap c\_2Eieee\_2Elszero V0a))))))))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(p V0A) \vee (p V1B)) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & (\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee ( \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (41)$$

### Theorem 1

$$(\forall V0a \in ty\_2Eieee\_2Efloat. (((\neg((p (ap c_2Eieee_2Elisnan \\ & V0a)) \wedge (p (ap c_2Eieee_2EInfinity V0a)))) \wedge ((\neg((p (ap c_2Eieee_2Elisnan \\ & V0a)) \wedge (p (ap c_2Eieee_2EFinite V0a)))) \wedge (\neg((p (ap c_2Eieee_2EInfinity \\ & V0a)) \wedge (p (ap c_2Eieee_2EFinite V0a)))))))$$