

thm_2Efloat_2EFLOAT__MUL (TMKXA_{pvLd-} cpmh9yx33vi5A5Crjm3GWh4HPB)

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Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Eieeee_2Efloat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieeee_2Efloat \tag{3}$$

Let $c_2Eieeee_2Edefloat : \iota$ be given. Assume the following.

$$c_2Eieeee_2Edefloat \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^{ty_2Eieeee_2Efloat}) \tag{4}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{6}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (8)$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (9)$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

Definition 8 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (ap (c_2Emin_2E_3D_3D_3E$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b))^{(2^{A_27b})^{A_27a}} \quad (10)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2EABS_prod$

Definition 13 We define $c_2Eieee_2Efloat_format$ to be $(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum$

Let $c_2Eieee_2Efraction : \iota$ be given. Assume the following.

$$c_2Eieee_2Efraction \in (ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum))} \quad (11)$$

Let $c_2Eieee_2Exponent : \iota$ be given. Assume the following.

$$c_2Eieee_2Exponent \in (ty_2Enum_2Enum)^{(ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum))} \quad (12)$$

Definition 14 We define $c_2Eieee_2Eis_zero$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 15 We define $c_2Eieee_2Elszero$ to be $\lambda V0a \in ty_2Eieee_2Efloat. (ap (ap c_2Eieee_2Eis_zero c_2Epair_2E_2C$

Definition 16 We define c_Ebool_EF to be $(ap (c_Ebool_E21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 17 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_E21\ 2))$.

Definition 18 We define $c_Eieeee_Eis_denormal$ to be $\lambda V0X \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)$.

Definition 19 We define $c_Eieeee_Eisdenormal$ to be $\lambda V0a \in ty_Eieeee_Efloat.(ap (ap c_Eieeee_Eis_denormal V0a))$.

Definition 20 We define $c_Ebool_E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21\ 2) (\lambda V2t \in 2.V2t))))$.

Let $c_Eieeee_Eexpwidth : \iota$ be given. Assume the following.

$$c_Eieeee_Eexpwidth \in (ty_Eenum_Eenum^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)}) \quad (13)$$

Let $c_Earithmetic_EEXP : \iota$ be given. Assume the following.

$$c_Earithmetic_EEXP \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (14)$$

Let $c_Earithmetic_E2D : \iota$ be given. Assume the following.

$$c_Earithmetic_E2D \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (15)$$

Definition 21 We define c_Eieeee_Eemax to be $\lambda V0X \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)$.

Definition 22 We define c_Emin_E40 to be $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 23 We define c_Ebool_E3F to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_Emin_E40 P a))))$.

Definition 24 We define $c_Eprim_rec_E3C$ to be $\lambda V0m \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum$.

Definition 25 We define $c_Eieeee_Eis_normal$ to be $\lambda V0X \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum_Eenum)$.

Definition 26 We define $c_Eieeee_Eisnormal$ to be $\lambda V0a \in ty_Eieeee_Efloat.(ap (ap c_Eieeee_Eis_normal V0a))$.

Definition 27 We define $c_Eieeee_EFinite$ to be $\lambda V0a \in ty_Eieeee_Efloat.(ap (ap c_Ebool_E5C_2F V0a))$.

Let $ty_Eieeee_ERoundmode : \iota$ be given. Assume the following.

$$nonempty\ ty_Eieeee_ERoundmode \quad (16)$$

Let $c_Eieeee_ETO_nearest : \iota$ be given. Assume the following.

$$c_Eieeee_ETO_nearest \in ty_Eieeee_ERoundmode \quad (17)$$

Let $ty_Erealax_EReal : \iota$ be given. Assume the following.

$$nonempty\ ty_Erealax_EReal \quad (18)$$

Let $c_2Eieeee_2Eround : \iota$ be given. Assume the following.

$$c_2Eieeee_2Eround \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ ty_2Erealax_2Ereal)\ ty_2Eieeee_2Eroundmode)\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum) \quad (19)$$

Let $c_2Eieeee_2Efloat : \iota$ be given. Assume the following.

$$c_2Eieeee_2Efloat \in (ty_2Eieeee_2Efloat\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ ty_2Erealax_2Ereal) \quad (20)$$

Let $c_2Eieeee_2Evalof : \iota$ be given. Assume the following.

$$c_2Eieeee_2Evalof \in ((ty_2Erealax_2Ereal\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ ty_2Erealax_2Ereal)\ ty_2Eieeee_2Evalof) \quad (21)$$

Definition 28 We define $c_2Eieeee_2Eval$ to be $\lambda V0a \in ty_2Eieeee_2Efloat.(ap\ (ap\ c_2Eieeee_2Evalof\ c_2Eieeee_2Evalof))\ a$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (22)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal)\ ty_2Erealax_2Ereal) \quad (23)$$

Definition 29 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ ty_2Erealax_2Ereal)\ a)$

Let $c_2Erealax_2Etreall_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal) \quad (24)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal)\ ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \quad (25)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal\ (2\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal)) \quad (26)$$

Definition 30 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ r$

Definition 31 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal)\ T1$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal)\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ ty_2Erealax_2Ereal) \quad (27)$$

Definition 32 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 33 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 34 We define $c_2Efloat_2Eerror$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap c_2Ereal_2Ereal_sub$

Let $c_2Eieeee_2Eis_valid : \iota$ be given. Assume the following.

$$c_2Eieeee_2Eis_valid \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)}))$$

(28)

Definition 35 We define $c_2Eieeee_2Eis_finite$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Let $c_2Eieeee_2Efracwidth : \iota$ be given. Assume the following.

$$c_2Eieeee_2Efracwidth \in (ty_2Enum_2Enum^{(ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)})$$

(29)

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum})$$

(30)

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal})$$

(31)

Let $c_2Erealax_2Etrealmul_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})$$

(32)

Definition 36 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Definition 37 We define $c_2Eieeee_2Ebias$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})$$

(33)

Definition 38 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 39 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 40 We define $c_2Eieeee_2Ethreshold$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})$$

(34)

Definition 41 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 42 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 43 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 44 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Definition 45 We define $c_2Eieeee_2Eminus_zero$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 46 We define $c_2Eieeee_2Eplus_zero$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 47 We define $c_2Eieeee_2Ezerosign$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 48 We define $c_2Eieeee_2Eminus_infinity$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 49 We define $c_2Eieeee_2Eplus_infinity$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Let $c_2Eieeee_2Esign : \iota$ be given. Assume the following.

$$c_2Eieeee_2Esign \in (ty_2Enum_2Enum^{(ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)} \quad (35)$$

Definition 50 We define $c_2Eieeee_2Eis_nan$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 51 We define $c_2Eieeee_2Esome_nan$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 52 We define $c_2Eieeee_2Elsnan$ to be $\lambda V0a \in ty_2Eieeee_2Efloat.(ap (ap c_2Eieeee_2Eis_nan c_2Eieeee_2Efloat$

Definition 53 We define $c_2Eieeee_2Eis_infinity$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 54 We define $c_2Eieeee_2EInfinity$ to be $\lambda V0a \in ty_2Eieeee_2Efloat.(ap (ap c_2Eieeee_2Eis_infinity$

Definition 55 We define $c_2Eieeee_2Efmul$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum$

Definition 56 We define $c_2Eieeee_2Efloat_mul$ to be $\lambda V0a \in ty_2Eieeee_2Efloat.\lambda V1b \in ty_2Eieeee_2Efloat$

Assume the following.

$$True \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (38)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& (p V0t)) \Leftrightarrow (p V0t)))))) \quad (40)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (41)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (42)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (43)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p V0t)))))) \quad (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t1 \in A.27a.(\forall V1t2 \in \\
& A.27a.(((ap (ap (ap (c_2Ebool_2ECOND A.27a) c_2Ebool_2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A.27a) c_2Ebool_2EF) \\
& V0t1) V1t2) = V1t2)))))) \quad (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\
& V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Eieeee_2Efloat. ((\neg((p (ap c_2Eieeee_2Elsnan \\
& V0a)) \wedge (p (ap c_2Eieeee_2EInfinity V0a)))) \wedge ((\neg((p (ap c_2Eieeee_2Elsnan \\
& V0a)) \wedge (p (ap c_2Eieeee_2EFinite V0a)))) \wedge ((\neg((p (ap c_2Eieeee_2EInfinity \\
& V0a)) \wedge (p (ap c_2Eieeee_2EFinite V0a)))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Eieeee_2Edefloat (ap \\
& c_2Eieeee_2Efloat (ap (ap (ap c_2Eieeee_2ERound c_2Eieeee_2Efloat_format) \\
& c_2Eieeee_2ETo_nearest) V0x))) = (ap (ap (ap c_2Eieeee_2ERound \\
& c_2Eieeee_2Efloat_format) c_2Eieeee_2ETo_nearest) V0x)))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1b \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Eieeee_2Evalof c_2Eieeee_2Efloat_format) (ap c_2Eieeee_2Edefloat \\
& (ap c_2Eieeee_2Efloat (ap (ap (ap c_2Eieeee_2Ezerosign c_2Eieeee_2Efloat_format) \\
& V1b) (ap (ap (ap c_2Eieeee_2ERound c_2Eieeee_2Efloat_format) c_2Eieeee_2ETo_nearest) \\
& V0x)))))) = (ap (ap c_2Eieeee_2Evalof c_2Eieeee_2Efloat_format) \\
& (ap (ap (ap c_2Eieeee_2ERound c_2Eieeee_2Efloat_format) c_2Eieeee_2ETo_nearest) \\
& V0x))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Eieeee_2Efloat. ((p (ap c_2Eieeee_2EFinite V0a)) \Leftrightarrow \\
& (p (ap (ap c_2Eieeee_2Eis_finite c_2Eieeee_2Efloat_format) (\\
& ap c_2Eieeee_2Edefloat V0a))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. (\forall V1x \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs V1x)) (ap \\
& c_2Eieeee_2Ethreshold c_2Eieeee_2Efloat_format))) \Rightarrow (p (ap (ap \\
& c_2Eieeee_2Eis_finite c_2Eieeee_2Efloat_format) (ap c_2Eieeee_2Edefloat \\
& (ap c_2Eieeee_2Efloat (ap (ap (ap c_2Eieeee_2Ezerosign c_2Eieeee_2Efloat_format) \\
& V0b) (ap (ap (ap c_2Eieeee_2ERound c_2Eieeee_2Efloat_format) c_2Eieeee_2ETo_nearest) \\
& V1x))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add V1y) (ap (ap c_2Ereal_2Ereal_sub V0x) V1y)) = V0x))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (64)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (65)$$

Theorem 1

$$\begin{aligned} & (\forall V0a \in ty_2Eieee_2Efloat. (\forall V1b \in ty_2Eieee_2Efloat. \\ & (((p (ap c_2Eieee_2EFinite V0a)) \wedge ((p (ap c_2Eieee_2EFinite V1b)) \wedge \\ & (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs (ap (ap c_2Erealax_2Ereal_mul \\ & (ap c_2Eieee_2EVal V0a)) (ap c_2Eieee_2EVal V1b)))) (ap c_2Eieee_2Ethreshold \\ & c_2Eieee_2Efloat_format)))))) \Rightarrow ((p (ap c_2Eieee_2EFinite (ap \\ & (ap c_2Eieee_2Efloat_mul V0a) V1b))) \wedge ((ap c_2Eieee_2EVal (ap \\ & (ap c_2Eieee_2Efloat_mul V0a) V1b)) = (ap (ap c_2Erealax_2Ereal_add \\ & (ap (ap c_2Erealax_2Ereal_mul (ap c_2Eieee_2EVal V0a)) (ap c_2Eieee_2EVal \\ & V1b))) (ap c_2Efloat_2Eerror (ap (ap c_2Erealax_2Ereal_mul (\\ & ap c_2Eieee_2EVal V0a)) (ap c_2Eieee_2EVal V1b)))))))))) \end{aligned}$$