thm_2Efloat_2EFLOAT__SUB (TMK74oyu1u9iRzUFACxGDTkXRcZLXKXnNuX)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Eieee\_2Efloat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eieee\_2Efloat \tag{3}$$

Let $c\_2Eieee\_2Edefloat : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Edefloat \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))^{ty\_2Eieee\_2Efloat}) \tag{4}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 3 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 4 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{8}$$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Definition 7 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{10}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 13 *We define* c_2Eieee_2Efloat__format *to be* $(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2E$

Let $c\_2Eieee\_2Efraction : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efraction \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2} \tag{11}$$

Let $c\_2Eieee\_2Eexponent : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexponent \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2} \tag{12}$$

Definition 14 *We define* c_2Eieee_2Eis__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 15 *We define* c_2Eieee_2EIszero *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap\ \mathsf{c\_2Eieee\_2Eis\_\_zero}\ \mathsf{c\_}$

Definition 16 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 18 *We define* c_2Eieee_2Eis__denormal *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Definition 19 *We define* c_2Eieee_2EIsdenormal *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap\ \mathsf{c\_2Eieee\_2Eis\_\_de}$

Definition 20 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Eieee\_2Eexpwidth : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexpwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{13}$$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

Definition 21 *We define* c_2Eieee_2Eemax *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enu$

Definition 22 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 23 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 24 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 25 *We define* c_2Eieee_2Eis__normal *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 26 *We define* c_2Eieee_2EIsnormal *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap\ \mathsf{c\_2Eieee\_2Eis\_\_norm}$

Definition 27 *We define* c_2Eieee_2EFinite *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_5C\_2F}\ (a$

Let $ty\_2Eieee\_2Eroundmode : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eieee\_2Eroundmode \tag{16}$$

Let $c\_2Eieee\_2ETo\_\_nearest : \iota$ be given. Assume the following.

$$c\_2Eieee\_2ETo\_\_nearest \in ty\_2Eieee\_2Eroundmode \tag{17}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{18}$$

Let $c\_2Eieee\_2Eround : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eround \in ((((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Epro\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))^{ty\_2Erealax\_2Ereal})^{ty\_2Eieee\_2Eroundmode})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{19}$$

Let $c\_2Eieee\_2Efloat : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efloat \in (ty\_2Eieee\_2Efloat^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enur} \tag{20}$$

Let $c\_2Eieee\_2Evalof : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Evalof \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{21}$$

Definition 28 *We define* c_2Eieee_2EVal *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap\ c\_2Eieee\_2Evalof$ c_2E

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{22}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{23}$$

Definition 29 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 (t

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{24}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{25}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \tag{26}$$

Definition 30 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ t$

Definition 31 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{27}$$

Definition 32 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 33 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Definition 34 *We define* c_2Efloat_2Eerror *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap$ c_2Ereal_2Ereal__sub

Let $c\_2Eieee\_2Eis\_\_valid : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eis\_\_valid \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2E} \tag{28}$$

Definition 35 *We define* c_2Eieee_2Eis__finite *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Let $c\_2Eieee\_2Efracwidth : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efracwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{29}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{30}$$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{31}$$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{32}$$

Definition 36 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Definition 37 *We define* c_2Eieee_2Ebias *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{33}$$

Definition 38 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 39 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Definition 40 *We define* c_2Eieee_2Ethreshold *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{34}$$

Definition 41 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 42 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 43 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 44 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ ($c_2Ebool_2ECONI

Let $c\_2Eieee\_2ETo\_\_ninfinity : \iota$ be given. Assume the following.

$$c\_2Eieee\_2ETo\_\_ninfinity \in ty\_2Eieee\_2Eroundmode \tag{35}$$

Definition 45 *We define* c_2Eieee_2Eminus__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Definition 46 *We define* c_2Eieee_2Eplus__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 47 *We define* c_2Eieee_2Ezerosign *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Let $c\_2Eieee\_2Esign : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Esign \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum} \tag{36}$$

Definition 48 *We define* c_2Eieee_2Eminus *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enu$

Definition 49 *We define* c_2Eieee_2Eis__nan *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2En$

Definition 50 *We define* c_2Eieee_2Esome__nan *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 51 *We define* c_2Eieee_2EIsnan *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap$ c_2Eieee_2Eis__nan c_2

Definition 52 *We define* c_2Eieee_2Eis__infinity *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 53 *We define* c_2Eieee_2EInfinity *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap$ c_2Eieee_2Eis__infinit

Definition 54 *We define* c_2Eieee_2Efsub *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum$

Definition 55 *We define* c_2Eieee_2Efloat__sub *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.\lambda V1b \in ty\_2Eieee\_2Eflo$

Assume the following.

$$True \tag{37}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{40}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{41}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{42}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{47}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27)))))))) \tag{49}$$

Assume the following.

$$(\forall V0a \in ty\_2Eieee\_2Efloat.((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EIsnan}\ V0a)) \wedge (p\ (ap\ \mathsf{c\_2Eieee\_2EInfinity}\ V0a)))) \wedge ((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EIsnan}\ V0a)) \wedge (p\ (ap\ \mathsf{c\_2Eieee\_2EFinite}\ V0a)))) \wedge (\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EInfinity}\ V0a)) \wedge (p\ (ap\ \mathsf{c\_2Eieee\_2EFinite}\ V0a))))))) \tag{50}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ c\_2Eieee\_2Edefloat\ (ap\ c\_2Eieee\_2Efloat\ (ap\ (ap\ (ap\ c\_2Eieee\_2Eround\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ c\_2Eieee\_2ETo\_\_nearest)\ V0x))) = (ap\ (ap\ (ap\ c\_2Eieee\_2Eround\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ c\_2Eieee\_2ETo\_\_nearest)\ V0x))) \tag{51}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Eieee\_2Evalof\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ (ap\ c\_2Eieee\_2Edefloat\ (ap\ c\_2Eieee\_2Efloat\ (ap\ (ap\ (ap\ \mathsf{c\_2Eieee\_2Ezerosign}\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ V1b)\ (ap\ (ap\ (ap\ c\_2Eieee\_2Eround\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ c\_2Eieee\_2ETo\_\_nearest)\ V0x))))) = (ap\ (ap\ c\_2Eieee\_2Evalof\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ (ap\ (ap\ (ap\ c\_2Eieee\_2Eround\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ c\_2Eieee\_2ETo\_\_nearest)\ V0x))))) \tag{52}$$

Assume the following.

$$(\forall V0a \in ty\_2Eieee\_2Efloat.((p\ (ap\ \mathsf{c\_2Eieee\_2EFinite}\ V0a)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eieee\_2Eis\_\_finite}\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ (ap\ c\_2Eieee\_2Edefloat\ V0a))))) \tag{53}$$

Assume the following.

$$(\forall V0b \in ty\_2Enum\_2Enum.(\forall V1x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Ereal\_2Eabs}\ V1x))\ (ap\ \mathsf{c\_2Eieee\_2Ethreshold}\ \mathsf{c\_2Eieee\_2Efloat\_\_format}))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eieee\_2Eis\_\_finite}\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ (ap\ c\_2Eieee\_2Edefloat\ (ap\ c\_2Eieee\_2Efloat\ (ap\ (ap\ (ap\ \mathsf{c\_2Eieee\_2Ezerosign}\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ V0b)\ (ap\ (ap\ (ap\ c\_2Eieee\_2Eround\ \mathsf{c\_2Eieee\_2Efloat\_\_format})\ c\_2Eieee\_2ETo\_\_nearest)\ V1x))))))))) \tag{54}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V0x)\ V1y)) = V0x))) \tag{55}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{56}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{58}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{62}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{63}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{64}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{65}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{66}$$

Theorem 1

$$(\forall V0a \in ty\_2Eieee\_2Efloat.(\forall V1b \in ty\_2Eieee\_2Efloat. (((p\ (ap\ \mathsf{c\_2Eieee\_2EFinite}\ V0a))\wedge((p\ (ap\ \mathsf{c\_2Eieee\_2EFinite}\ V1b))\wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Ereal\_2Eabs}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ \mathsf{c\_2Eieee\_2EVal}\ V0a))\ (ap\ \mathsf{c\_2Eieee\_2EVal}\ V1b))))\ (ap\ \mathsf{c\_2Eieee\_2Ethreshold}\ \mathsf{c\_2Eieee\_2Efloat\_\_format})))))\Rightarrow((p\ (ap\ \mathsf{c\_2Eieee\_2EFinite}\ (ap\ (ap\ \mathsf{c\_2Eieee\_2Efloat\_\_sub}\ V0a)\ V1b)))\wedge((ap\ \mathsf{c\_2Eieee\_2EVal}\ (ap\ (ap\ \mathsf{c\_2Eieee\_2Efloat\_\_sub}\ V0a)\ V1b)) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ \mathsf{c\_2Eieee\_2EVal}\ V0a))\ (ap\ \mathsf{c\_2Eieee\_2EVal}\ V1b)))\ (ap\ \mathsf{c\_2Efloat\_2Eerror}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ \mathsf{c\_2Eieee\_2EVal}\ V0a))\ (ap\ \mathsf{c\_2Eieee\_2EVal}\ V1b)))))))))$$