

thm_2Efloat_2EFLOAT__SUB__RELATIVE
 (TMJfLzod-
 cHU5mVnWh9rXJMyQo4AoKsHeJSB)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Eieeee_2Efloat : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieeee_2Efloat \tag{3}$$

Let $c_2Eieeee_2Edefloat : \iota$ be given. Assume the following.

$$c_2Eieeee_2Edefloat \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))^{ty_2Eieeee_2Efloat}) \tag{4}$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{6}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Definition 16 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Definition 17 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2E_2F } V0t)))$

Definition 18 We define `c_2Eieeee_2Eis__denormal` to be $\lambda V0X \in (ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum \ ty_2Enum)$

Definition 19 We define `c_2Eieeee_2Elsdenormal` to be $\lambda V0a \in ty_2Eieeee_2Efloat.(\text{ap } (\text{ap } (\text{c_2Eieeee_2Eis_de } a)))$

Definition 20 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))))$

Let `c_2Eieeee_2Eexpwidth` : ι be given. Assume the following.

$$c_2Eieeee_2Eexpwidth \in (ty_2Enum_2Enum^{(ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum \ ty_2Enum \ ty_2Enum \ ty_2Enum)}) \quad (13)$$

Let `c_2Earithmetic_2EEXP` : ι be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum \ ty_2Enum})^{ty_2Enum \ ty_2Enum}) \quad (14)$$

Let `c_2Earithmetic_2E_2D` : ι be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum \ ty_2Enum})^{ty_2Enum \ ty_2Enum}) \quad (15)$$

Definition 21 We define `c_2Eieeee_2Eemax` to be $\lambda V0X \in (ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum \ ty_2Enum)$

Definition 22 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p \ (\text{ap } P \ x)) \ \text{then } (\text{the } (\lambda x.x \in A \ \lambda y.y \in A) \ \text{of type } \iota \Rightarrow \iota)$.

Definition 23 We define `c_2Ebool_2E_3F` to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(\text{ap } V0P \ (\text{ap } (\text{c_2Emin_2E_40 } a))))$

Definition 24 We define `c_2Eprim__rec_2E_3C` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define `c_2Eieeee_2Eis__normal` to be $\lambda V0X \in (ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum \ ty_2Enum)$

Definition 26 We define `c_2Eieeee_2Elsnormal` to be $\lambda V0a \in ty_2Eieeee_2Efloat.(\text{ap } (\text{ap } (\text{c_2Eieeee_2Eis_norm } a))))$

Definition 27 We define `c_2Eieeee_2EFinite` to be $\lambda V0a \in ty_2Eieeee_2Efloat.(\text{ap } (\text{ap } (\text{c_2Ebool_2E_5C_2F } a))))$

Definition 28 We define `c_2Eieeee_2Ebias` to be $\lambda V0X \in (ty_2Epair_2Eprod \ ty_2Enum_2Enum \ ty_2Enum \ ty_2Enum)$

Let `ty_2Ehreal_2Ehreal` : ι be given. Assume the following.

$$\text{nonempty } ty_2Ehreal_2Ehreal \quad (16)$$

Let `ty_2Erealax_2Ereal` : ι be given. Assume the following.

$$\text{nonempty } ty_2Erealax_2Ereal \quad (17)$$

Let `c_2Erealax_2Ereal__REP__CLASS` : ι be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (18)$$

Definition 29 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$
Let $c_2Erealax_2Etreall_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (19)$$

Let $c_2Erealax_2Etreall_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal)}) \quad (20)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})}) \quad (21)$$

Definition 30 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 31 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (22)$$

Definition 32 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Etreall_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (23)$$

Definition 33 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (24)$$

Definition 34 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 35 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Eieeee_2Efracwidth : \iota$ be given. Assume the following.

$$c_2Eieeee_2Efracwidth \in (ty_2Eenum_2Eenum^{(ty_2Epair_2Eprod ty_2Eenum_2Eenum ty_2Eenum_2Eenum)}) \quad (25)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}) \quad (26)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (27)$$

Let $c_2Erealax_2Etreal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (28)$$

Definition 36 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Definition 37 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 38 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 39 We define $c_2Eieeee_2Ethreshold$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum$

Definition 40 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a$

Definition 41 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Definition 42 We define $c_2Efloat_2Enormalizes$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ c_2Ebool_2E_2F$

Let $c_2Eieeee_2Eis_valid : \iota$ be given. Assume the following.

$$c_2Eieeee_2Eis_valid \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum$$
))}) \quad (29)

Definition 43 We define $c_2Eieeee_2Eis_finite$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum$

Let $ty_2Eieeee_2Eroundmode : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eieeee_2Eroundmode \quad (30)$$

Let $c_2Eieeee_2Eround : \iota$ be given. Assume the following.

$$c_2Eieeee_2Eround \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Erealax_2Ereal})^{ty_2Eieeee_2Eroundmode})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum$$
)) \quad (31)

Let $c_2Eieeee_2ETO_ninfinitiy : \iota$ be given. Assume the following.

$$c_2Eieeee_2ETO_ninfinitiy \in ty_2Eieeee_2Eroundmode \quad (32)$$

Definition 44 We define $c_2Eieeee_2Eminus_zero$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum$

Definition 45 We define $c_2Eieeee_2Eplus_zero$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum$

Definition 46 We define $c_2Eieeee_2Ezerosign$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum$

Let $c_Eieee_Esign : \iota$ be given. Assume the following.

$$c_Eieee_Esign \in (ty_Eenum_Eenum^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ (ty_Epair_Eprod\ ty_Eenum_Eenum))}) \quad (33)$$

Definition 47 We define c_Eieee_Eminus to be $\lambda V0X \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum)$

Definition 48 We define $c_Eieee_Eis_nan$ to be $\lambda V0X \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum)$

Definition 49 We define $c_Eieee_Esome_nan$ to be $\lambda V0X \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum)$

Let $c_Eieee_Eevalof : \iota$ be given. Assume the following.

$$c_Eieee_Eevalof \in ((ty_Erealax_Ereal^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ (ty_Epair_Eprod\ ty_Eenum_Eenum))}) \quad (34)$$

Definition 50 We define c_Eieee_Eeval to be $\lambda V0a \in ty_Eieee_Efloat.(ap\ (ap\ c_Eieee_Eevalof\ c_Eenum))$

Definition 51 We define c_Eieee_Eisnan to be $\lambda V0a \in ty_Eieee_Efloat.(ap\ (ap\ c_Eieee_Eis_nan\ c_Eenum))$

Definition 52 We define $c_Eieee_Eis_infinity$ to be $\lambda V0X \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum)$

Definition 53 We define $c_Eieee_Einfinity$ to be $\lambda V0a \in ty_Eieee_Efloat.(ap\ (ap\ c_Eieee_Eis_infinity\ c_Eenum))$

Let $c_Eieee_ETO_nearest : \iota$ be given. Assume the following.

$$c_Eieee_ETO_nearest \in ty_Eieee_Eroundmode \quad (35)$$

Definition 54 We define c_Eieee_Efsub to be $\lambda V0X \in (ty_Epair_Eprod\ ty_Eenum_Eenum\ ty_Eenum)$

Let $c_Eieee_Efloat : \iota$ be given. Assume the following.

$$c_Eieee_Efloat \in (ty_Eieee_Efloat^{(ty_Epair_Eprod\ ty_Eenum_Eenum\ (ty_Epair_Eprod\ ty_Eenum_Eenum))}) \quad (36)$$

Definition 55 We define $c_Eieee_Efloat_sub$ to be $\lambda V0a \in ty_Eieee_Efloat.\lambda V1b \in ty_Eieee_Efloat$

Definition 56 We define $c_Eprim_rec_EPRE$ to be $\lambda V0m \in ty_Eenum_Eenum.(ap\ (ap\ (ap\ (c_Ebool_Ebool\ c_Eenum))))$

Let $c_Eenumerals_Ehelp : \iota$ be given. Assume the following.

$$c_Eenumerals_Ehelp \in ((ty_Eenum_Eenum^{ty_Eenum_Eenum})^{ty_Eenum_Eenum}) \quad (37)$$

Let $c_Earithmetic_EODD : \iota$ be given. Assume the following.

$$c_Earithmetic_EODD \in (2^{ty_Eenum_Eenum}) \quad (38)$$

Assume the following.

$$True \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (45)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (47)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V0t1) V1t2) = V1t2)))) \quad (51)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (52)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) V5y_27)))))) \quad (53)$$

Assume the following.

$$(\forall V0a \in ty_2Eieee_2Efloat. (((\neg((p (ap c_2Eieee_2Elsnan V0a)) \wedge (p (ap c_2Eieee_2EInfinity V0a)))) \wedge ((\neg((p (ap c_2Eieee_2Elsnan V0a)) \wedge (p (ap c_2Eieee_2EFinite V0a)))) \wedge (\neg((p (ap c_2Eieee_2EInfinity V0a)) \wedge (p (ap c_2Eieee_2EFinite V0a)))))) \quad (54)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal. ((ap c_2Eieee_2Edefloat (ap c_2Eieee_2Efloat (ap (ap (ap c_2Eieee_2ERound c_2Eieee_2Efloat_format) c_2Eieee_2ETO_nearest) V0x))) = (ap (ap (ap c_2Eieee_2ERound c_2Eieee_2Efloat_format) c_2Eieee_2ETO_nearest) V0x))) \quad (55)$$

Assume the following.

$$(\forall V0x \in ty_2Erealx_2Ereal. (\forall V1b \in ty_2Enum_2Enum. (((ap (ap c_2Eieee_2Evalof c_2Eieee_2Efloat_format) (ap c_2Eieee_2Edefloat (ap c_2Eieee_2Efloat (ap (ap (ap c_2Eieee_2Ezerosign c_2Eieee_2Efloat_format) V1b) (ap (ap (ap c_2Eieee_2ERound c_2Eieee_2Efloat_format) c_2Eieee_2ETO_nearest) V0x)))))) = (ap (ap c_2Eieee_2Evalof c_2Eieee_2Efloat_format) (ap (ap (ap c_2Eieee_2ERound c_2Eieee_2Efloat_format) c_2Eieee_2ETO_nearest) V0x)))))) \quad (56)$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Eieee_2Efloat. ((p (ap c_2Eieee_2EFinite V0a)) \Leftrightarrow \\
& \quad (p (ap (ap c_2Eieee_2Eis_finite c_2Eieee_2Efloat_format) (\\
& \quad \quad ap c_2Eieee_2Edefloat V0a)))))) \tag{57}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((p (ap c_2Efloat_2Enormalizes \\
& \quad V0x)) \Rightarrow (\exists V1e \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Ereal_2Ereal_lte \\
& \quad (ap c_2Ereal_2Eabs V1e)) (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& \quad (ap (ap c_2Ereal_2Epow (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& \quad (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))))))) \wedge \\
& \quad ((ap c_2Eieee_2EVal (ap c_2Eieee_2Efloat (ap (ap (ap c_2Eieee_2ERound \\
& \quad c_2Eieee_2Efloat_format) c_2Eieee_2ETO_nearest) V0x))) = \\
& \quad (ap (ap c_2Erealax_2Ereal_mul V0x) (ap (ap c_2Erealax_2Ereal_add \\
& \quad (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V1e)))))) \tag{58}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty_2Enum_2Enum. (\forall V1x \in ty_2Erealax_2Ereal. \\
& \quad ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Eabs V1x)) (ap \\
& \quad c_2Eieee_2Ethreshold c_2Eieee_2Efloat_format))) \Rightarrow (p (ap (ap \\
& \quad c_2Eieee_2Eis_finite c_2Eieee_2Efloat_format) (ap c_2Eieee_2Edefloat \\
& \quad (ap c_2Eieee_2Efloat (ap (ap (ap c_2Eieee_2Ezerosign c_2Eieee_2Efloat_format) \\
& \quad V0b) (ap (ap (ap c_2Eieee_2ERound c_2Eieee_2Efloat_format) c_2Eieee_2ETO_nearest \\
& \quad V1x)))))))))) \tag{59}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap c_2Eprim_rec_2EPRE c_2Earithmetic_2EZERO) = c_2Earithmetic_2EZERO) \wedge \\
& \quad (((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) = \\
& \quad \quad c_2Earithmetic_2EZERO) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap \\
& \quad c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\
& \quad V0n))) = (ap c_2Earithmetic_2EBIT2 (ap c_2Eprim_rec_2EPRE (ap \\
& \quad c_2Earithmetic_2EBIT1 V0n)))))) \wedge ((\forall V1n \in ty_2Enum_2Enum. \\
& \quad ((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT2 \\
& \quad V1n))) = (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 \\
& \quad V1n)))) \wedge (\forall V2n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2EBIT2 V2n)) = (ap c_2Earithmetic_2EBIT1 V2n)))))) \tag{60}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0acc \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (((ap (ap c_2Enumeral_2Etxp_help c_2Earithmetic_2EZERO) V0acc) = \\
& (ap c_2Earithmetic_2EBIT2 V0acc)) \wedge (((ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Earithmetic_2EBIT1 V1n)) V0acc) = (ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 V1n)) (ap \\
& c_2Earithmetic_2EBIT1 V0acc))) \wedge ((ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Earithmetic_2EBIT2 V1n)) V0acc) = (ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Earithmetic_2EBIT1 V1n)) (ap c_2Earithmetic_2EBIT1 V0acc))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) \wedge (((ap (ap c_2Earithmetic_2EEXP (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V0n))) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Enumeral_2Etxp_help \\
& (ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2EBIT1 V0n))) c_2Earithmetic_2EZERO))) \wedge \\
& ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V0n))) = (ap c_2Earithmetic_2ENUMERAL \\
& (ap (ap c_2Enumeral_2Etxp_help (ap c_2Earithmetic_2EBIT1 V0n)) \\
& c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2a \in ty_2Enum_2Enum. (\forall V3y \in ty_2Erealax_2Ereal. \\
& (((ap (ap c_2Ereal_2Epow V0x) c_2Enum_2E0) = (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& (((ap (ap c_2Ereal_2Epow (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V1n))) = \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \wedge (((ap (ap c_2Ereal_2Epow \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 V1n))) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) \wedge (((ap (ap c_2Ereal_2Epow (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2a)) (ap c_2Earithmetic_2ENUMERAL \\
& V1n)) = (ap c_2Ereal_2Ereal_of_num (ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL V2a)) (ap c_2Earithmetic_2ENUMERAL \\
& V1n)))))) \wedge (((ap (ap c_2Ereal_2Epow (ap c_2Erealax_2Ereal_neg \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL V2a)))) \\
& (ap c_2Earithmetic_2ENUMERAL V1n)) = (ap (ap (ap (ap (c_2Ebool_2ECOND \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}) (ap c_2Earithmetic_2EODD \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) c_2Erealax_2Ereal_neg \\
& (\lambda V4x \in ty_2Erealax_2Ereal.V4x)) (ap c_2Ereal_2Ereal_of_num \\
& (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V2a)) \\
& (ap c_2Earithmetic_2ENUMERAL V1n)))))) \wedge ((ap (ap c_2Ereal_2Epow \\
& (ap (ap c_2Ereal_2E_2F V0x) V3y)) V1n) = (ap (ap c_2Ereal_2E_2F (\\
& ap (ap c_2Ereal_2Epow V0x) V1n)) (ap (ap c_2Ereal_2Epow V3y) V1n))))))))))
\end{aligned} \tag{63}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{64}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{67}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False))) \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{73}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{78}$$

Theorem 1

$$\begin{aligned} & (\forall V0a \in ty_2Eieee_2Efloat. (\forall V1b \in ty_2Eieee_2Efloat. \\ & (((p (ap c_2Eieee_2EFinite V0a)) \wedge ((p (ap c_2Eieee_2EFinite V1b)) \wedge \\ & (p (ap c_2Efloat_2Enormalizes (ap (ap c_2Ereal_2Ereal_sub (ap \\ & c_2Eieee_2EVal V0a)) (ap c_2Eieee_2EVal V1b)))))) \Rightarrow ((p (ap c_2Eieee_2EFinite \\ & (ap (ap c_2Eieee_2Efloat_sub V0a) V1b))) \wedge (\exists V2e \in ty_2Erealax_2Ereal. \\ & ((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Eabs V2e)) (ap (\\ & ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) (ap (ap \\ & c_2Ereal_2Epow (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 (ap c_2Earithmetic_2EBIT1 (ap c_2Earithmetic_2EBIT1 \\ & (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))))))))) \wedge \\ & ((ap c_2Eieee_2EVal (ap (ap c_2Eieee_2Efloat_sub V0a) V1b)) = \\ & (ap (ap c_2Erealax_2Ereal_mul (ap (ap c_2Ereal_2Ereal_sub (\\ & ap c_2Eieee_2EVal V0a)) (ap c_2Eieee_2EVal V1b))) (ap (ap c_2Erealax_2Ereal_add \\ & (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\ & ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V2e))))))))) \end{aligned}$$