

thm_2Efloat_2EIS__FINITE__FINITE (TM-
ceTG3jVRR24jHKNrdKAk7bZy4UHNqXP5Q)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 3 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 4 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a}) A_27b) A_27c) A_27b)$

Definition 5 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}) A_27b) A_27c) A_27b))$

Definition 7 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in (A_27b^{A_27c}). \lambda V1g$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^\omega) \quad (3)$$

Definition 8 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow & nonempty\ (ty_2Epair_2Eprod \\ & A0\ A1) \end{aligned} \quad (4)$$

Let $c_2Eieee_2Efraction : \iota$ be given. Assume the following.

$$c_2Eieee_2Efraction \in (ty_2Enum_2Enum^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum))})$$
(5)

Let $c_2Eieee_2Eexponent : \iota$ be given. Assume the following.

$$c_2Eieee_2Eexponent \in (ty_2Enum_2Enum^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum))})$$
(6)

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 11 We define $c_2Eieee_2Eis_zero$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum)$

Definition 12 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E))$

Definition 14 We define $c_2Eieee_2Eis_denormal$ to be $\lambda V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum)$

Definition 15 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum})$$
(7)

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega})$$
(8)

Definition 16 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$
(9)

Definition 17 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B n) 1)$

Definition 18 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Eieee_2Eexpwidth : \iota$ be given. Assume the following.

$$c_2Eieee_2Eexpwidth \in (ty_2Enum_2Enum^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})$$
(10)

Definition 19 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B n) 0)$

Let $c_2Earithmetic_2EXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 20 We define c_2Eieee_2Eemax to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum$

Definition 21 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 22 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 23 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 24 We define $c_2Eieee_2Eis_normal$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum$

Let $c_2Eieee_2Eis_valid : \iota$ be given. Assume the following.

$$c_2Eieee_2Eis_valid \in ((2^{(ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))})^{(ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))}) \quad (13)$$

Definition 25 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Definition 26 We define $c_2Eieee_2Eis_finite$ to be $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (14)$$

Definition 27 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (15)$$

Definition 28 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Definition 29 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2$

Definition 30 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2$

Definition 31 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 32 We define $c_2E_{pred_set_2EFINITE}$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}).(ap (c_2Ebool_2E_21 (2$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (17)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p V0t) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p (ap V1P V3x)) \vee (p V0Q)))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \quad (26) \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow ((p V3y_27))))))) \end{aligned}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2El \\ A_27a) V0x) = V0x)) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\ & \forall V0f \in (A_27b^{A_27a}).(((ap (ap (c_2Ecombin_2El A_27a A_27b \\ A_27b) (c_2Ecombin_2El A_27b)) V0f) = V0f) \wedge ((ap (ap (c_2Ecombin_2El \\ A_27a A_27b A_27a) V0f) (c_2Ecombin_2El A_27a)) = V0f))) \quad (28) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & (p (ap (c_2Epred_set_2EFINITE (ty_2Epair_2Eprod ty_2Enum_2Enum \\ (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Epair_2Eprod ty_2Enum_2Enum)) (ap (c_2Epred_set_2EGSPEC \\ (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum \\ ty_2Enum_2Enum)) (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod \\ ty_2Enum_2Enum ty_2Enum_2Enum))) (ty_2Epair_2Eprod \\ ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)). \\ (ap (ap (c_2Epair_2E2C (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod \\ ty_2Enum_2Enum ty_2Enum_2Enum)) 2) V1a) (ap (ap c_2Eieee_2Eis_valid \\ V0X) V1a))))))) \quad (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\ & \forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in \\ & A_27b.(((ap (ap (c_2Epair_2E2C A_27a A_27b) V0x) V1y) = (ap (ap \\ (c_2Epair_2E2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \\ & \forall V0f \in ((ty_2Epair_2Eprod A_27a 2)^{A_27b}).(\forall V1v \in \\ & A_27a.((p (ap (ap (c_2Ebool_2EIN A_27a) V1v) (ap (c_2Epred_set_2EGSPEC \\ A_27a A_27b) V0f))) \Leftrightarrow (\exists V2x \in A_27b.((ap (ap (c_2Epair_2E2C \\ A_27a 2) V1v) c_2Ebool_2ET) = (ap V0f V2x)))))) \quad (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).((p\ (ap\ c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \Rightarrow (\forall V1t \in (2^{A_27a}). \\ ((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V1t)\ V0s)) \Rightarrow (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1t))))))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow \text{False}))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p\ V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p\ V1B)) \Rightarrow \text{False})))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow \text{False})))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow \text{False}) \Rightarrow (((p\ V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (37)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow \\ &(p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ &((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow \\ &(p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ &((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow \\ &(p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ &((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p))))))))))) \end{aligned} \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee \\ (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (43)$$

Theorem 1

$$\begin{aligned} & (\forall V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum). \\ & \quad (p (ap (c_2Epred_set_2EFINITE (ty_2Epair_2Eprod ty_2Enum_2Enum \\ & \quad (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum)))) (ap (c_2Epred_set_2EGSPEC \\ & \quad (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum \\ & \quad ty_2Enum_2Enum)) (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod \\ & \quad ty_2Enum_2Enum ty_2Enum_2Enum)))) (\lambda V1a \in (ty_2Epair_2Eprod \\ & \quad ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2Enum_2Enum))). \\ & \quad (ap (ap (c_2Epair_2E_2C (ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod \\ & \quad ty_2Enum_2Enum ty_2Enum_2Enum)) 2) V1a) (ap (ap c_2Eieee_2Eis_finite \\ & \quad V0X) V1a)))))) \end{aligned}$$