



Let  $c\_Efrac : \iota$  be given. Assume the following.

$$c\_Efrac \in (ty\_Enum\_Enum^{(ty\_Epair\_Eprod\ ty\_Enum\_Enum\ (ty\_Epair\_Eprod\ ty\_Enum\_Enum))}) \quad (5)$$

Let  $c\_Eexp : \iota$  be given. Assume the following.

$$c\_Eexp \in (ty\_Enum\_Enum^{(ty\_Epair\_Eprod\ ty\_Enum\_Enum\ (ty\_Epair\_Eprod\ ty\_Enum\_Enum))}) \quad (6)$$

**Definition 9** We define  $c\_E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 10** We define  $c\_E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_E21\ 2)\ (\lambda V2t \in 2.V0t))\ V1t2))\ V0t1$ .

**Definition 11** We define  $c\_Eis\_zero$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)$ .

**Definition 12** We define  $c\_E2F$  to be  $(ap\ (c\_E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 13** We define  $c\_E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_E3D\_3D\_3E\ V0t)\ c\_E21\ 2)\ V0t)$ .

**Definition 14** We define  $c\_Eis\_denormal$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)$ .

**Definition 15** We define  $c\_E2ZERO$  to be  $c\_E0$ .

Let  $c\_EREP\_num : \iota$  be given. Assume the following.

$$c\_EREP\_num \in (\omega^{ty\_Enum\_Enum}) \quad (7)$$

Let  $c\_ESUC\_REP : \iota$  be given. Assume the following.

$$c\_ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

**Definition 16** We define  $c\_ESUC$  to be  $\lambda V0m \in ty\_Enum\_Enum.(ap\ c\_EABS\_num\ m)$ .

Let  $c\_E2B : \iota$  be given. Assume the following.

$$c\_E2B \in ((ty\_Enum\_Enum^{(ty\_Enum\_Enum)}\ ty\_Enum\_Enum)) \quad (9)$$

**Definition 17** We define  $c\_E2BIT1$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap\ (ap\ c\_E2B\ n))\ V0n$ .

**Definition 18** We define  $c\_E2ENUMERAL$  to be  $\lambda V0x \in ty\_Enum\_Enum.V0x$ .

Let  $c\_Eexpwidth : \iota$  be given. Assume the following.

$$c\_Eexpwidth \in (ty\_Enum\_Enum^{(ty\_Epair\_Eprod\ ty\_Enum\_Enum\ ty\_Enum\_Enum)}) \quad (10)$$

**Definition 19** We define  $c\_E2BIT2$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap\ (ap\ c\_E2B\ n))\ V0n$ .



**Definition 32** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c\_2Ebool\_2E\_21 (2$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (17)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (21)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A-27a}).((\forall V2x \in A\_27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p (ap V1P V3x)) \vee (p V0Q)))))) \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (25)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27)))) \quad (26)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap \ (c\_2Ecombin\_2EI \ A\_27a) \ V0x) = V0x)) \quad (27)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}). (((ap \ (ap \ (c\_2Ecombin\_2Eo \ A\_27a \ A\_27b \ A\_27b) \ (c\_2Ecombin\_2EI \ A\_27b)) \ V0f) = V0f) \wedge ((ap \ (ap \ (c\_2Ecombin\_2Eo \ A\_27a \ A\_27b \ A\_27a) \ V0f) \ (c\_2Ecombin\_2EI \ A\_27a)) = V0f))) \quad (28)$$

Assume the following.

$$(\forall V0X \in (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum). \\ (p \ (ap \ (c\_2Epred\_set\_2EFINITE \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)) \ (ap \ (c\_2Epred\_set\_2EGSPEC \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)) \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)) \ (\lambda V1a \in (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum). \\ (ap \ (ap \ (c\_2Epair\_2E\_2C \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)) \ 2) \ V1a) \ (ap \ (ap \ c\_2Eieee\_2Eis\_valid \ V0X) \ V1a)))))) \quad (29)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in A\_27b. (((ap \ (ap \ (c\_2Epair\_2E\_2C \ A\_27a \ A\_27b) \ V0x) \ V1y) = (ap \ (ap \ (c\_2Epair\_2E\_2C \ A\_27a \ A\_27b) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (30)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow ( \\ \forall V0f \in ((ty\_2Epair\_2Eprod \ A\_27a \ 2)^{A\_27b}). (\forall V1v \in A\_27a. ((p \ (ap \ (ap \ (c\_2Ebool\_2EIN \ A\_27a) \ V1v) \ (ap \ (c\_2Epred\_set\_2EGSPEC \ A\_27a \ A\_27b) \ V0f))) \Leftrightarrow (\exists V2x \in A\_27b. ((ap \ (ap \ (c\_2Epair\_2E\_2C \ A\_27a \ 2) \ V1v) \ c\_2Ebool\_2ET) = (ap \ V0f \ V2x)))))) \quad (31)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). ((p\ (ap \\ & \quad (c\_2Epred\_set\_2EFINITE\ A\_27a)\ V0s)) \Rightarrow (\forall V1t \in (2^{A\_27a}). \\ & ((p\ (ap\ (ap\ (c\_2Epred\_set\_2ESUBSET\ A\_27a)\ V1t)\ V0s)) \Rightarrow (p\ (ap\ (c\_2Epred\_set\_2EFINITE \\ & \quad A\_27a)\ V1t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg( \\ & p\ V2r)) \vee (\neg(p\ V1q)))))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge (( \\ & \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee \\ & (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))) \end{aligned} \quad (41)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (43)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\ & (p\ (ap\ (c\_2Epred\_set\_2EFINITE\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\ & (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)))\ (ap\ (c\_2Epred\_set\_2EGSPEC \\ & (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\ & ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\ & ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)))\ (\lambda V1a \in (ty\_2Epair\_2Eprod \\ & ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))). \\ & (ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\ & ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ 2)\ V1a)\ (ap\ (ap\ c\_2Eieeee\_2Eis\_finite \\ & V0X)\ V1a)))))) \end{aligned}$$