

# thm\_2Efloat\_2EIS\_VALID\_CLOSEST (TMat- MvEKev71BbidSEiiV453RVQm5mUbXop)

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**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E21$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ecombin\_2E21$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}))$

**Definition 4** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 5** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A.\lambda A\_27a : \iota.(ap (ap (c\_2Ecombin\_2E21 A\_27a (A\_27a^{A\_27a})) A\_27a))$

**Definition 6** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E3D (2^{A\_27a})) P)))$

**Definition 7** We define  $c\_2Ecombin\_2Eo$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g \in (A\_27a^{A\_27c}).f g)$

**Definition 8** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 9** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21 2) (V0t1 V1t2))))$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{2}$$

**Definition 10** We define  $c\_2Epair\_2E2C$  to be  $\lambda A.\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2EABS\_prod x y)))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (3)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (4)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (5)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (6)$$

**Definition 11** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ .if  $(\exists x \in A.p\ (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 12** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal$ .(ap  $(c\_2Emin\_2E\_40$  (t

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (7)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (9)$$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal$ .(ap  $c\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (10)$$

**Definition 15** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal$ . $\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 16** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal$ . $\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{11}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{12}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{13}$$

**Definition 17** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{14}$$

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \tag{15}$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 19** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 20** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E7E))$

**Definition 21** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 22** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ V1f\ V0x)))$

**Definition 23** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND)$

**Definition 24** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A-27a}).(ap\ V1f\ V0x)))$

**Definition 25** We define  $c\_2Eieeee\_2Eis\_closest$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Erealax\_2Ereal^{A-27a}).\lambda V1s \in ($

**Definition 26** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E40)$

**Definition 27** We define  $c\_2Eieeee\_2Eclosest$  to be  $\lambda A\_27a : \iota.\lambda V0v \in (ty\_2Erealax\_2Ereal^{A-27a}).\lambda V1p \in ($

Let  $c\_2Eieeee\_2Efraction : \iota$  be given. Assume the following.

$$c\_2Eieeee\_2Efraction \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum))}) \tag{16}$$

Let  $c\_2Eieeee\_2Eexponent : \iota$  be given. Assume the following.

$$c\_2Eieeee\_2Eexponent \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum))}) \tag{17}$$

**Definition 28** We define  $c\_Eis\_zero$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum)$

**Definition 29** We define  $c\_Eis\_denormal$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum)$

**Definition 30** We define  $c\_EArithmetic\_EZERO$  to be  $c\_Eenum\_E0$ .

Let  $c\_Eenum\_EERP\_num : \iota$  be given. Assume the following.

$$c\_Eenum\_EERP\_num \in (\omega^{ty\_Eenum\_Eenum}) \quad (18)$$

Let  $c\_Eenum\_EESUC\_REP : \iota$  be given. Assume the following.

$$c\_Eenum\_EESUC\_REP \in (\omega^{omega}) \quad (19)$$

**Definition 31** We define  $c\_Eenum\_EESUC$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.(ap\ c\_Eenum\_EABS\_num)$

Let  $c\_EArithmetic\_E2B : \iota$  be given. Assume the following.

$$c\_EArithmetic\_E2B \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (20)$$

**Definition 32** We define  $c\_EArithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_Eenum\_Eenum.(ap\ (ap\ c\_EArithmetic\_EBIT1))$

**Definition 33** We define  $c\_EArithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_Eenum\_Eenum.V0x$ .

Let  $c\_Eieeee\_Eexpwidth : \iota$  be given. Assume the following.

$$c\_Eieeee\_Eexpwidth \in (ty\_Eenum\_Eenum^{(ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum\_Eenum)}) \quad (21)$$

**Definition 34** We define  $c\_EArithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_Eenum\_Eenum.(ap\ (ap\ c\_EArithmetic\_EBIT2))$

Let  $c\_EArithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_EArithmetic\_EEXP \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (22)$$

Let  $c\_EArithmetic\_E2D : \iota$  be given. Assume the following.

$$c\_EArithmetic\_E2D \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (23)$$

**Definition 35** We define  $c\_Eieeee\_Eemax$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum)$

**Definition 36** We define  $c\_Eprim\_rec\_E3C$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

**Definition 37** We define  $c\_Eieeee\_Eis\_normal$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum)$

Let  $c\_Eieeee\_Eis\_valid : \iota$  be given. Assume the following.

$$c\_Eieeee\_Eis\_valid \in ((2^{(ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ (ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum\_Eenum))}) \quad (24)$$

**Definition 38** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

**Definition 39** We define `c_2Eieeee_2Eis__finite` to be  $\lambda V0X \in (ty_2Epair_2Eprod ty_2Enum_2Enum ty_2E$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (28)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((ap (c_2Ecombin_2EI A_27a) V0x) = V0x)) \quad (33)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (c\_2Ecombin\_2Eo\ A\_27a\ A\_27b \\ & A\_27b)\ (c\_2Ecombin\_2EI\ A\_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c\_2Ecombin\_2Eo \\ & A\_27a\ A\_27b\ A\_27a)\ V0f)\ (c\_2Ecombin\_2EI\ A\_27a)) = V0f))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\ & (\forall V1v \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))} \\ & (\forall V2p \in (2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))} \\ & (\forall V3x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ c\_2Eieeee\_2Eis\_finite \\ & V0X)\ (ap\ (ap\ (ap\ (ap\ (c\_2Eieeee\_2Eclosest\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\ & (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ V1v)\ V2p) \\ & (ap\ (c\_2Epred\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\ & (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod \\ & ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) \\ & (\lambda V4a \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\ & ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)).(ap\ (ap\ (c\_2Epair\_2E\_2C\ (ty\_2Epair\_2Eprod \\ & ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) \\ & 2)\ V4a)\ (ap\ (ap\ c\_2Eieeee\_2Eis\_finite\ V0X)\ V4a))))))\ V3x)))))) \end{aligned} \quad (35)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\ & (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{46}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0X \in (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)). \\
& (\forall V1v \in (ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum))} \\
& (\forall V2p \in (2^{(ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum))} \\
& (\forall V3x \in ty\_2Erealax\_2Ereal. (p \ (ap \ (ap \ c\_2Eieeee\_2Eis\_valid \\
& V0X) \ (ap \ (ap \ (ap \ (ap \ (c\_2Eieeee\_2Eclosest \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \\
& (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum))) \ V1v) \ V2p) \\
& (ap \ (c\_2Epred\_set\_2EGSPEC \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \\
& (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)) \ (ty\_2Epair\_2Eprod \\
& ty\_2Enum\_2Enum \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum))) \\
& (\lambda V4a \in (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ (ty\_2Epair\_2Eprod \\
& ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)). (ap \ (ap \ (c\_2Epair\_2E2C \ (ty\_2Epair\_2Eprod \\
& ty\_2Enum\_2Enum \ (ty\_2Epair\_2Eprod \ ty\_2Enum\_2Enum \ ty\_2Enum\_2Enum)) \\
& 2) \ V4a) \ (ap \ (ap \ c\_2Eieeee\_2Eis\_finite \ V0X) \ V4a)))))) \ V3x))))))
\end{aligned}$$