thm_2Efloat_2EIS__VALID__ROUND (TMXTvYocKL5SwKS4mHhkiLsa16WHfb9rnu5)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{5}$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 11 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{7}$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Definition 13 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 14 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 15 *We define* c_2Eieee_2Eminus__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Definition 16 *We define* c_2Eieee_2Eplus__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Let $c\_2Eieee\_2Eis\_\_valid : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eis\_\_valid \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2E} \tag{9}$$

Let $ty\_2Eieee\_2Eroundmode : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eieee\_2Eroundmode \tag{10}$$

Let $c\_2Eieee\_2ETo\_\_ninfinity : \iota$ be given. Assume the following.

$$c\_2Eieee\_2ETo\_\_ninfinity \in ty\_2Eieee\_2Eroundmode \tag{11}$$

Let $c\_2Eieee\_2ETo\_\_pinfinity : \iota$ be given. Assume the following.

$$c\_2Eieee\_2ETo\_\_pinfinity \in ty\_2Eieee\_2Eroundmode \tag{12}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{13}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{14}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{15}$$

Definition 17 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 18 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{16}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{17}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{18}$$

Definition 19 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ t$

Definition 20 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{19}$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{20}$$

Definition 21 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 22 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 23 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 24 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Eieee\_2Efracwidth : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efracwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (21)$$

Definition 25 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (22)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (23)$$

Let $c\_2Eieee\_2Eexpwidth : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexpwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (24)$$

Definition 26 *We define* c_2Eieee_2Eemax *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enu$

Definition 27 *We define* c_2Eieee_2Etopfloat *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 28 *We define* c_2Ereal_2Ereal__gt *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ere$

Definition 29 *We define* c_2Eieee_2Ebottomfloat *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (25)$$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (26)$$

Definition 30 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (27)$$

Definition 31 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 32 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 33 *We define* c_2Eieee_2Ebias *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (28)$$

Definition 34 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 35 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Definition 36 *We define* c_2Eieee_2Elargest *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2En$

Let $c\_2Eieee\_2Efloat\_\_To\_\_zero : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efloat\_\_To\_\_zero \in ty\_2Eieee\_2Eroundmode \quad (29)$$

Let $c\_2Eieee\_2Efraction : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efraction \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \quad (30)$$

Let $c\_2Eieee\_2Eexponent : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexponent \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2} \quad (31)$$

Definition 37 *We define* c_2Eieee_2Eis__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 38 *We define* c_2Eieee_2Eis__denormal *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Definition 39 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 40 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 41 *We define* c_2Eieee_2Eis__normal *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 42 *We define* c_2Eieee_2Eis__finite *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (32)$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (33)$$

Let $c\_2Eieee\_2Evalof : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Evalof \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \quad (34)$$

Definition 43 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a:\iota.(\lambda V0x\in A\_27a.(\lambda V1f\in(2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 44 *We define* c_2Eieee_2Eis__closest *to be* $\lambda A\_27a:\iota.\lambda V0v\in(ty\_2Erealax\_2Ereal^{A\_27a}).\lambda V1s$

Definition 45 *We define* c_2Eieee_2Eclosest *to be* $\lambda A\_27a:\iota.\lambda V0v\in(ty\_2Erealax\_2Ereal^{A\_27a}).\lambda V1p\in($

Definition 46 *We define* c_2Eieee_2Eplus__infinity *to be* $\lambda V0X\in(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Definition 47 *We define* c_2Ereal_2Ereal__ge *to be* $\lambda V0x\in ty\_2Erealax\_2Ereal.\lambda V1y\in ty\_2Erealax\_2Er$

Definition 48 *We define* c_2Eieee_2Eminus__infinity *to be* $\lambda V0X\in(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$

Definition 49 *We define* c_2Eieee_2Ethreshold *to be* $\lambda V0X\in(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Let $c\_2Eieee\_2ETo\_\_nearest:\iota$ be given. Assume the following.

$$c\_2Eieee\_2ETo\_\_nearest\in ty\_2Eieee\_2Eroundmode \tag{35}$$

Let $c\_2Eieee\_2Eround:\iota$ be given. Assume the following.

$$c\_2Eieee\_2Eround\in((((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Epro\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))^{ty\_2Erealax\_2Ereal})^{ty\_2Eieee\_2Eroundmode})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{36}$$

Assume the following.

$$True \tag{37}$$

Assume the following.

$$(\forall V0t1\in 2.(\forall V1t2\in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{38}$$

Assume the following.

$$(\forall V0t\in 2.(False\Rightarrow(p\ V0t))) \tag{39}$$

Assume the following.

$$(\forall V0t\in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{40}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow(p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t1\in A\_27a.(\forall V1t2\in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2)=V0t1)\wedge((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2)=V1t2)))) \tag{42}$$

Assume the following.

$$(\forall V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum).
((p\ (ap\ (ap\ c_2Eieee_2Eis__valid\ V0X)\ (ap\ \mathsf{c_2Eieee_2Eminus__infinity}\ V0X)))\wedge((p\ (ap\ (ap\ c_2Eieee_2Eis__valid\ V0X)\ (ap\ \mathsf{c_2Eieee_2Eplus__infinity}\ V0X)))\wedge((p\ (ap\ (ap\ c_2Eieee_2Eis__valid\ V0X)\ (ap\ \mathsf{c_2Eieee_2Etopfloat}\ V0X)))\wedge((p\ (ap\ (ap\ c_2Eieee_2Eis__valid\ V0X)\ (ap\ \mathsf{c_2Eieee_2Ebottomfloat}\ V0X)))\wedge((p\ (ap\ (ap\ c_2Eieee_2Eis__valid\ V0X)\ (ap\ \mathsf{c_2Eieee_2Eplus__zero}\ V0X)))\wedge(p\ (ap\ (ap\ c_2Eieee_2Eis__valid\ V0X)\ (ap\ \mathsf{c_2Eieee_2Eminus__zero}\ V0X)))))))) \quad (43)$$

Assume the following.

$$(\forall V0X \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum).
(\forall V1v \in (ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum}
(\forall V2p \in (2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))})
(\forall V3x \in ty_2Erealax_2Ereal.(p\ (ap\ (ap\ c_2Eieee_2Eis__valid\ V0X)\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c_2Eieee_2Eclosest}\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))\ V1v)\ V2p)\ (ap\ (c_2Epred__set_2EGSPEC\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)))\ (\lambda V4a \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)).(ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum))\ 2)\ V4a)\ (ap\ (ap\ \mathsf{c_2Eieee_2Eis__finite}\ V0X)\ V4a)))))\ V3x))))))) \quad (44)$$

Assume the following.

$$
\begin{gathered}
((\forall V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). \\
(\forall V1x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ (ap\ c\_2Eieee\_2Eround \\
V0X)\ c\_2Eieee\_2ETo\_\_nearest)\ V1x) = (ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND} \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum)))\ (ap\ (ap\ \textsf{c\_2Ereal\_2Ereal\_\_lte}\ V1x)\ (ap\ \textsf{c\_2Erealax\_2Ereal\_\_neg} \\
(ap\ \textsf{c\_2Eieee\_2Ethreshold}\ V0X))))\ (ap\ \textsf{c\_2Eieee\_2Eminus\_\_infinity} \\
V0X))\ (ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)))\ (ap\ (ap\ \textsf{c\_2Ereal\_2Ereal\_\_ge} \\
V1x)\ (ap\ \textsf{c\_2Eieee\_2Ethreshold}\ V0X)))\ (ap\ \textsf{c\_2Eieee\_2Eplus\_\_infinity} \\
V0X))\ (ap\ (ap\ (ap\ (ap\ (\textsf{c\_2Eieee\_2Eclosest}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)))\ (ap\ c\_2Eieee\_2Evalof \\
V0X))\ (\lambda V2a \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)).(ap\ c\_2Earithmetic\_2EEVEN \\
(ap\ c\_2Eieee\_2Efraction\ V2a))))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (\\
ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum))\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)))\ (\lambda V3a \in (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
(ap\ (ap\ (\textsf{c\_2Epair\_2E\_2C}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ 2)\ V3a)\ (ap\ (ap\ \textsf{c\_2Eieee\_2Eis\_\_finite} \\
V0X)\ V3a)))))\ V1x))))))\wedge((\forall V4X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum).(\forall V5x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ (\\
ap\ c\_2Eieee\_2Eround\ V4X)\ c\_2Eieee\_2Efloat\_\_To\_\_zero)\ V5x) = (\\
ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)))\ (ap\ (ap\ \textsf{c\_2Erealax\_2Ereal\_\_lt} \\
V5x)\ (ap\ \textsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \textsf{c\_2Eieee\_2Elargest}\ V4X)))) \\
(ap\ \textsf{c\_2Eieee\_2Ebottomfloat}\ V4X))\ (ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND} \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum)))\ (ap\ (ap\ \textsf{c\_2Ereal\_2Ereal\_\_gt}\ V5x)\ (ap\ \textsf{c\_2Eieee\_2Elargest} \\
V4X)))\ (ap\ \textsf{c\_2Eieee\_2Etopfloat}\ V4X))\ (ap\ (ap\ (ap\ (ap\ (\textsf{c\_2Eieee\_2Eclosest} \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum)))\ (ap\ c\_2Eieee\_2Evalof\ V4X))\ (\lambda V6x \in (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
\textsf{c\_2Ebool\_2ET}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum)))\ (\lambda V7a \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)).(ap\ (ap\ (\\
\textsf{c\_2Epair\_2E\_2C}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ 2)\ V7a)\ (ap\ (ap\ \textsf{c\_2Ebool\_2E\_2F\_5C} \\
(ap\ (ap\ \textsf{c\_2Eieee\_2Eis\_\_finite}\ V4X)\ V7a))\ (ap\ (ap\ \textsf{c\_2Ereal\_2Ereal\_\_lte} \\
(ap\ \textsf{c\_2Ereal\_2Eabs}\ (ap\ (ap\ c\_2Eieee\_2Evalof\ V4X)\ V7a)))\ (ap\ \textsf{c\_2Ereal\_2Eabs} \\
V5x)))))))\ V5x))))))\wedge((\forall V8X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum).(\forall V9x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ (\\
ap\ c\_2Eieee\_2Eround\ V8X)\ c\_2Eieee\_2ETo\_\_pinfinity)\ V9x) = (ap \\
(ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND}\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)))\ (ap\ (ap\ \textsf{c\_2Erealax\_2Ereal\_\_lt} \\
V9x)\ (ap\ \textsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \textsf{c\_2Eieee\_2Elargest}\ V8X)))) \\
(ap\ \textsf{c\_2Eieee\_2Ebottomfloat}\ V8X))\ (ap\ (ap\ (ap\ (\textsf{c\_2Ebool\_2ECOND} \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum)))\ (ap\ (ap\ \textsf{c\_2Ereal\_2Ereal\_\_gt}\ V9x)\ (ap\ \textsf{c\_2Eieee\_2Elargest} \\
V8X)))\ (ap\ \textsf{c\_2Eieee\_2Eplus\_\_infinity}\ V8X))\ (ap\ (ap\ (ap\ (ap\ (\textsf{c\_2Eieee\_2Eclosest} \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum)))\ (ap\ c\_2Eieee\_2Evalof\ V8X))\ (\lambda V10x \in (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)). \\
\textsf{c\_2Ebool\_2ET}))\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ (ty\_2Epair\_2Eprod \\
ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)) \\
(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum \\
ty\_2Enum\_2Enum)))\ (\lambda V11a \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum
\end{gathered}
$$

Theorem 1

$$(\forall V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum). (\forall V1x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ c\_2Eieee\_2Eis\_\_valid\ V0X)\ (ap\ (ap\ (ap\ c\_2Eieee\_2Eround\ V0X)\ c\_2Eieee\_2ETo\_\_nearest)\ V1x)))))$$