thm_2Efloat_2EVALOF (TMEjgk3KnEcpEDhJDXGsJM7u5Z4roNVNazx)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $c\_2Eieee\_2Esign : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Esign \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum} \tag{3}$$

Let $c\_2Eieee\_2Eexponent : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexponent \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2} \tag{4}$$

Let $c\_2Eieee\_2Efraction : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efraction \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{5}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{7}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{8}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{9}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{10}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{11}$$

Definition 10 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 11 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Eieee\_2Efracwidth : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efracwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{12}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{13}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{14}$$

Definition 12 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 13 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (15)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (16)$$

Definition 14 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (17)$$

Definition 15 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 16 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Eieee\_2Eexpwidth : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexpwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (18)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (19)$$

Definition 17 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (20)$$

Definition 18 *We define* c_2Eieee_2Ebias *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (21)$$

Definition 19 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (22)$$

Definition 20 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 21 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{23}$$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{24}$$

Definition 22 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{25}$$

Definition 23 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Eieee\_2Evalof : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Evalof \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{26}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{27}$$

Definition 24 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 25 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Assume the following.

$$True \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{30}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. (\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg (p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27)))))))) \tag{32}$$

Assume the following.

$$(\forall V0s \in ty\_2Enum\_2Enum.(\forall V1e \in ty\_2Enum\_2Enum.(\forall V2f \in ty\_2Enum\_2Enum.((ap\ c\_2Eieee\_2Esign\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ V0s)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V1e)\ V2f))) = V0s)))) \tag{33}$$

Assume the following.

$$(\forall V0s \in ty\_2Enum\_2Enum.(\forall V1e \in ty\_2Enum\_2Enum.(\forall V2f \in ty\_2Enum\_2Enum.((ap\ c\_2Eieee\_2Eexponent\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ V0s)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V1e)\ V2f))) = V1e)))) \tag{34}$$

Assume the following.

$$(\forall V0s \in ty\_2Enum\_2Enum.(\forall V1e \in ty\_2Enum\_2Enum.(\forall V2f \in ty\_2Enum\_2Enum.((ap\ c\_2Eieee\_2Efraction\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))\ V0s)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V1e)\ V2f))) = V2f)))) \tag{35}$$

Assume the following.

$$
\begin{gathered}
(\forall V0X \in (\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Enum\_2Enum}\ \mathit{ty\_2Enum\_2Enum}). \\
(\forall V1s \in \mathit{ty\_2Enum\_2Enum}.(\forall V2e \in \mathit{ty\_2Enum\_2Enum}.(\\
\forall V3f \in \mathit{ty\_2Enum\_2Enum}.((ap\ (ap\ \mathit{c\_2Eieee\_2Evalof}\ V0X)\ (ap \\
(ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Enum\_2Enum}\ (\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Enum\_2Enum} \\
\mathit{ty\_2Enum\_2Enum}))\ V1s)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Enum\_2Enum} \\
\mathit{ty\_2Enum\_2Enum})\ V2e)\ V3f))) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ \mathit{ty\_2Erealax\_2Ereal}) \\
(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ \mathit{ty\_2Enum\_2Enum})\ V2e)\ \mathsf{c\_2Enum\_2E0}))\ (ap \\
(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap \\
(ap\ \mathit{c\_2Ereal\_2Epow}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num} \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \\
V1s))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (ap \\
\mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
(ap\ (ap\ \mathit{c\_2Ereal\_2Epow}\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ \mathsf{c\_2Eieee\_2Ebias} \\
V0X)))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num} \\
V3f))\ (ap\ (ap\ \mathit{c\_2Ereal\_2Epow}\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (ap \\
\mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
(ap\ \mathit{c\_2Eieee\_2Efracwidth}\ V0X)))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul} \\
(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathit{c\_2Ereal\_2Epow}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg} \\
(ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\\
ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V1s))\ (\\
ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ (ap\ \mathit{c\_2Ereal\_2Epow}\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num} \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
V2e))\ (ap\ (ap\ \mathit{c\_2Ereal\_2Epow}\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (ap \\
\mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
(ap\ \mathsf{c\_2Eieee\_2Ebias}\ V0X)))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (\\
ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap \\
\mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F} \\
(ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ V3f))\ (ap\ (ap\ \mathit{c\_2Ereal\_2Epow}\ (ap \\
\mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2} \\
\mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ \mathit{c\_2Eieee\_2Efracwidth}\ V0X)))))))))))
\end{gathered}
\tag{36}
$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a {\Rightarrow} \forall A\_27b.nonempty\ A\_27b {\Rightarrow} (\\
\forall V0x \in (\mathit{ty\_2Epair\_2Eprod}\ A\_27a\ A\_27b).(\exists V1q \in A\_27a. \\
(\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b) \\
V1q)\ V2r)))))
\end{gathered}
\tag{37}
$$

Theorem 1

$$
\begin{gathered}
(\forall \mathit{V0X} \in (\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Enum\_2Enum}\ \mathit{ty\_2Enum\_2Enum}). \\
(\forall \mathit{V1a} \in (\mathit{ty\_2Epair\_2Eprod}\ \mathit{ty\_2Enum\_2Enum}\ (\mathit{ty\_2Epair\_2Eprod} \\
\mathit{ty\_2Enum\_2Enum}\ \mathit{ty\_2Enum\_2Enum})).((\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Eieee\_2Evalof}\ \mathit{V0X}) \\
\mathit{V1a}) = (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2Ebool\_2ECOND}\ \mathit{ty\_2Erealax\_2Ereal})\ (\mathit{ap}\ (\mathit{ap} \\
(\mathsf{c\_2Emin\_2E\_3D}\ \mathit{ty\_2Enum\_2Enum})\ (\mathit{ap}\ \mathit{c\_2Eieee\_2Eexponent}\ \mathit{V1a})) \\
\mathsf{c\_2Enum\_2E0}))\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_mul} \\
(\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Epow}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \\
(\mathit{ap}\ \mathit{c\_2Eieee\_2Esign}\ \mathit{V1a})))\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Ereal\_2E\_2F}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
(\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Epow}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (\mathit{ap}\ \mathsf{c\_2Eieee\_2Ebias} \\
\mathit{V0X})))))\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Ereal\_2E\_2F}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num} \\
(\mathit{ap}\ \mathit{c\_2Eieee\_2Efraction}\ \mathit{V1a})))\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Epow}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
(\mathit{ap}\ \mathit{c\_2Eieee\_2Efracwidth}\ \mathit{V0X})))))\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_mul} \\
(\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Epow}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_neg} \\
(\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\\
\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ (\mathit{ap}\ \mathit{c\_2Eieee\_2Esign} \\
\mathit{V1a})))\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Ereal\_2E\_2F}\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Epow}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
(\mathit{ap}\ \mathit{c\_2Eieee\_2Eexponent}\ \mathit{V1a})))\ (\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Epow}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \\
(\mathit{ap}\ \mathsf{c\_2Eieee\_2Ebias}\ \mathit{V0X})))))\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (\\
\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (\mathit{ap} \\
\mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (\mathit{ap}\ (\mathit{ap}\ \mathsf{c\_2Ereal\_2E\_2F} \\
(\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (\mathit{ap}\ \mathit{c\_2Eieee\_2Efraction}\ \mathit{V1a}))) \\
(\mathit{ap}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Epow}\ (\mathit{ap}\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (\mathit{ap}\ \mathit{c\_2Eieee\_2Efracwidth} \\
\mathit{V0X})))))))))
\end{gathered}
$$