

# thm\_2Efloat\_2EVAL\_\_THRESHOLD (TMZVAus- RdKBHCSvG1azihg2QU4PjhQtB7Mn)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Eieeee\_2Efloat : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eieeee\_2Efloat \tag{3}$$

Let  $c\_2Eieeee\_2Edefloat : \iota$  be given. Assume the following.

$$c\_2Eieeee\_2Edefloat \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))^{ty\_2Eieeee\_2Efloat}) \tag{4}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{6}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{8}$$

**Definition 4** We define `c_2Ebool_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define `c_2Ebool_2E_21` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

**Definition 6** We define `c_2Enum_2ESUC` to be  $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS\_num ($

Let `c_2Earithmetic_2E_2B` :  $\iota$  be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

**Definition 7** We define `c_2Earithmetic_2EBIT2` to be  $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

**Definition 8** We define `c_2Earithmetic_2EBIT1` to be  $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

**Definition 9** We define `c_2Earithmetic_2ENUMERAL` to be  $\lambda V0x \in ty_2Enum_2Enum.V0x$ .

**Definition 10** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t))$

Let `c_2Epair_2EABS\_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS\_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (10)$$

**Definition 12** We define `c_2Epair_2E_2C` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2EABS\_prod$

**Definition 13** We define `c_2Eieee_2Efloat\_format` to be  $(ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum))$

Let `ty_2Erealax_2Ereal` :  $\iota$  be given. Assume the following.

$$nonempty ty_2Erealax_2Ereal \quad (11)$$

Let `c_2Eieee_2Evalof` :  $\iota$  be given. Assume the following.

$$c_2Eieee_2Evalof \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod ty_2Enum_2Enum (ty_2Epair_2Eprod ty_2Enum_2Enum))})^{ty_2Enum_2Enum}) \quad (12)$$

**Definition 14** We define `c_2Eieee_2EVal` to be  $\lambda V0a \in ty_2Eieee_2Efloat. (ap (ap c_2Eieee_2Evalof c_2Epair_2E_2C$

Let `ty_2Ehreal_2Ehreal` :  $\iota$  be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \quad (13)$$

Let `c_2Erealax_2Ereal\_REP\_CLASS` :  $\iota$  be given. Assume the following.

$$c_2Erealax_2Ereal\_REP\_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (14)$$



**Definition 25** We define  $c\_Eis\_zero$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum)$

**Definition 26** We define  $c\_Elszero$  to be  $\lambda V0a \in ty\_Efloat.(ap\ (ap\ c\_Eis\_zero\ c\_Eenum))$

**Definition 27** We define  $c\_Eis\_denormal$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum)$

**Definition 28** We define  $c\_Elsdenormal$  to be  $\lambda V0a \in ty\_Efloat.(ap\ (ap\ c\_Eis\_denormal\ c\_Eenum))$

**Definition 29** We define  $c\_Ebool\_E5C\_E2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_Ebool\_E21\ 2)\ (\lambda V2t \in 2))))$

Let  $c\_Eexpwidth : \iota$  be given. Assume the following.

$$c\_Eexpwidth \in (ty\_Eenum\_Eenum^{(ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum\_Eenum)}) \quad (22)$$

Let  $c\_Earithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEXP \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (23)$$

Let  $c\_Earithmetic\_E2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E2D \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (24)$$

**Definition 30** We define  $c\_Eemax$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum)$

**Definition 31** We define  $c\_Ebool\_E3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_Emin\_E40\ c\_Eenum))))$

**Definition 32** We define  $c\_Eprim\_rec\_E3C$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.\lambda V1n \in ty\_Eenum\_Eenum$

**Definition 33** We define  $c\_Eis\_normal$  to be  $\lambda V0X \in (ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum)$

**Definition 34** We define  $c\_Elsnormal$  to be  $\lambda V0a \in ty\_Efloat.(ap\ (ap\ c\_Eis\_normal\ c\_Eenum))$

**Definition 35** We define  $c\_Efinite$  to be  $\lambda V0a \in ty\_Efloat.(ap\ (ap\ c\_Ebool\_E5C\_E2F\ (ap\ c\_Eenum)))$

Let  $c\_Efracwidth : \iota$  be given. Assume the following.

$$c\_Efracwidth \in (ty\_Eenum\_Eenum^{(ty\_Epair\_Eprod\ ty\_Eenum\_Eenum\ ty\_Eenum\_Eenum)}) \quad (25)$$

Let  $c\_Ereal\_Epow : \iota$  be given. Assume the following.

$$c\_Ereal\_Epow \in ((ty\_Erealax\_Ereal^{ty\_Eenum\_Eenum})^{ty\_Erealax\_Ereal}) \quad (26)$$

Let  $c\_Erealax\_Etrealm\_Einv : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_Einv \in ((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}) \quad (27)$$

**Definition 36** We define  $c\_Erealax\_Einv$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal\_EABS)$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (28)$$

**Definition 37** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 38** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 39** We define  $c\_2Eieeee\_2Ebias$  to be  $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Eenum\_2Eenum\ ty\_2Eenum\_2Eenum)$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)} \quad (29)$$

**Definition 40** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 41** We define  $c\_2Ereal\_2E2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 42** We define  $c\_2Eieeee\_2Ethreshold$  to be  $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Eenum\_2Eenum\ ty\_2Eenum\_2Eenum)$

**Definition 43** We define  $c\_2Eieeee\_2Elargest$  to be  $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Eenum\_2Eenum\ ty\_2Eenum\_2Eenum)$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Eenum\_2Eenum}) \quad (30)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Eenum\_2Eenum}) \quad (31)$$

**Definition 44** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum.$

**Definition 45** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum.$

**Definition 46** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.\lambda V1n \in ty\_2Eenum\_2Eenum.$

**Definition 47** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Eenum\_2Eenum.(ap\ (ap\ (ap\ (c\_2Ebool\_2Ebool)))$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Eenum\_2Eenum^{ty\_2Eenum\_2Eenum})^{ty\_2Eenum\_2Eenum})^{ty\_2Eenum\_2Eenum} \quad (32)$$

**Definition 48** We define  $c\_2Eenumerals\_2EiZ$  to be  $\lambda V0x \in ty\_2Eenum\_2Eenum.V0x.$

Assume the following.

$$True \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Eieee\_2Efloat. ((p (ap c\_2Eieee\_2EFinite V0a)) \Rightarrow \\ & (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs (ap c\_2Eieee\_2EVal \\ & V0a))) (ap c\_2Eieee\_2Elargest c\_2Eieee\_2Efloat\_format)))))) \end{aligned} \quad (37)$$







Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) (ap c\_2Earithmic\_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) \\
& (ap c\_2Earithmic\_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))) \wedge (((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT1 V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1m) V0n))) \wedge ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& (ap c\_2Earithmic\_2EBIT2 V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) V1m))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\
& (\forall V2z \in ty\_2Erealx\_2Ereal. (((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0x) V1y)) \wedge (p (ap (ap c\_2Erealx\_2Ereal\_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c\_2Erealx\_2Ereal\_lt V0x) V2z))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap c\_2Erealx\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0n)) (ap c\_2Ereal\_2Ereal\_of\_num V1m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0n) V1m))) \wedge (((p (ap (ap c\_2Erealx\_2Ereal\_lt (ap c\_2Erealx\_2Ereal\_neg \\
& (ap c\_2Ereal\_2Ereal\_of\_num V0n)) (ap c\_2Ereal\_2Ereal\_of\_num \\
& V1m))) \Leftrightarrow ((\neg (V0n = c\_2Enum\_2E0)) \vee (\neg (V1m = c\_2Enum\_2E0)))) \wedge ((( \\
& p (ap (ap c\_2Erealx\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0n)) (ap c\_2Erealx\_2Ereal\_neg (ap c\_2Ereal\_2Ereal\_of\_num \\
& V1m)))) \Leftrightarrow False) \wedge ((p (ap (ap c\_2Erealx\_2Ereal\_lt (ap c\_2Erealx\_2Ereal\_neg \\
& (ap c\_2Ereal\_2Ereal\_of\_num V0n)) (ap c\_2Erealx\_2Ereal\_neg \\
& (ap c\_2Ereal\_2Ereal\_of\_num V1m)))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V1m) V0n))))))
\end{aligned} \tag{43}$$

### Theorem 1

$$\begin{aligned}
& (\forall V0a \in ty\_2Eieeee\_2Efloat. ((p (ap c\_2Eieeee\_2EFinite V0a)) \Rightarrow \\
& (p (ap (ap c\_2Erealx\_2Ereal\_lt (ap c\_2Ereal\_2Eabs (ap c\_2Eieeee\_2EVal \\
& V0a))) (ap c\_2Eieeee\_2Ethreshold c\_2Eieeee\_2Efloat\_format))))))
\end{aligned}$$