thm_2Efloat_2EZERO__NOT__NAN (TMZ-zvPUU2GD42bDG46UBumRM9J4gQEFY29o)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Eieee\_2Efloat : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eieee\_2Efloat \tag{3}$$

Let $c\_2Eieee\_2Edefloat : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Edefloat \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum))^{ty\_2Eieee\_2Efloat}) \tag{4}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{5}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{6}$$

Definition 3 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 4 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{8}$$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Definition 7 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{10}$$

Definition 12 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 13 *We define* c_2Eieee_2Efloat__format *to be* $(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2E$

Let $c\_2Eieee\_2Efraction : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efraction \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2E} \tag{11}$$

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $c\_2Eieee\_2Eexponent : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexponent \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2} \tag{12}$$

Definition 16 *We define* c_2Eieee_2Eis__denormal *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Definition 17 *We define* c_2Eieee_2Elsdenormal *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap$ c_2Eieee_2Eis__de

Let $c\_2Eieee\_2Eexpwidth : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Eexpwidth \in (ty\_2Enum\_2Enum^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{13}$$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

Definition 18 *We define* c_2Eieee_2Eemax *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enu$

Definition 19 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 20 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 21 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 22 *We define* c_2Eieee_2Eis__normal *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 23 *We define* c_2Eieee_2Elsnormal *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap$ c_2Eieee_2Eis__norm

Definition 24 *We define* c_2Eieee_2Eis__infinity *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 25 *We define* c_2Eieee_2EInfinity *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap$ c_2Eieee_2Eis__infinit

Definition 26 *We define* c_2Eieee_2Eis__nan *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2En$

Definition 27 *We define* c_2Eieee_2Elsnan *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap$ c_2Eieee_2Eis__nan c_2

Definition 28 *We define* c_2Eieee_2Eminus__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty$

Let $c\_2Eieee\_2Efloat : \iota$ be given. Assume the following.

$$c\_2Eieee\_2Efloat \in (ty\_2Eieee\_2Efloat^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enu} \tag{16}$$

Definition 29 *We define* c_2Eieee_2EMinus__zero *to be* $(ap\ c\_2Eieee\_2Efloat\ (ap$ c_2Eieee_2Eminus__zero c

Definition 30 *We define* c_2Eieee_2Eplus__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 31 *We define* c_2Eieee_2EPlus__zero *to be* $(ap\ c\_2Eieee\_2Efloat\ (ap$ c_2Eieee_2Eplus__zero c_2E

Definition 32 *We define* c_2Eieee_2Eis__zero *to be* $\lambda V0X \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2E$

Definition 33 *We define* c_2Eieee_2Elszero *to be* $\lambda V0a \in ty\_2Eieee\_2Efloat.(ap\ (ap$ c_2Eieee_2Eis__zero c_

Definition 34 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Assume the following.

$$True \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))))) \tag{21}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{24}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))))) \tag{25}$$

Assume the following.

$$(\forall V0a \in ty\_2Eieee\_2Efloat.((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EIsnan}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EInfinity}\ V0a))))\wedge((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EIsnan}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIsnormal}\ V0a))))\wedge((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EIsnan}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIsdenormal}\ V0a))))\wedge((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EIsnan}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIszero}\ V0a))))\wedge((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EInfinity}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIsnormal}\ V0a))))\wedge((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EInfinity}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIsdenormal}\ V0a))))\wedge((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EInfinity}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIszero}\ V0a))))\wedge((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EIsnormal}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIsdenormal}\ V0a))))\wedge((\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EIsnormal}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIszero}\ V0a))))\wedge(\neg((p\ (ap\ \mathsf{c\_2Eieee\_2EIsdenormal}\ V0a))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIszero}\ V0a)))))))))))))) \tag{26}$$

Assume the following.

$$((p\ (ap\ \mathsf{c\_2Eieee\_2EIszero}\ \mathsf{c\_2Eieee\_2EPlus\_\_zero}))\wedge(p\ (ap\ \mathsf{c\_2Eieee\_2EIszero}\ \mathsf{c\_2Eieee\_2EMinus\_\_zero}))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{31}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{32}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{33}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))) \tag{34}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \tag{35}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \tag{36}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{37}$$

Theorem 1

$$((\neg(p\ (ap\ \mathsf{c\_2Eieee\_2EIsnan}\ \mathsf{c\_2Eieee\_2EPlus\_\_zero}))) \wedge (\neg(p\ (ap\ \mathsf{c\_2Eieee\_2EIsnan}\ \mathsf{c\_2Eieee\_2EMinus\_\_zero}))))$$