

thm_2Efmamal_2EFAPPLY__fmap__CONS (TMU4MLC_{ssRP2jTgVb9myTeouZV6B2PFi9jj})

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x)$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_27a}))))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V0t \in 2. V0t)$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_27E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t)) (\text{c_2Ebool_2E_2F})))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 8 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 9 We define `c_2Ebool_2ECOND` to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (V1t1 \wedge V2t2))))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 A1) \tag{1}$$

Let `c_2Epair_2EABS__prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Epair_2EABS_prod } A_27a A_27b \in ((\text{ty_2Epair_2Eprod } A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 10 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (\text{ap } (\text{c_2Ebool_2E_27E } (V0x \wedge V1y)))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (3)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (4)$$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2Efmap\ A0\ A1) \quad (5)$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \quad (6)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL\ A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \quad (7)$$

Definition 11 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFAPPLY\ A_27a\ A_27b)$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b \in (((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)}) \quad (8)$$

Definition 12 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (9)$$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $c_2Efinite_map_2Efmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_ABS\ A_27a\ A_27b \in ((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a}}) \quad (10)$$

Definition 14 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota.(ap\ (c_2Efinite_map_2EFEMPTY\ A_27a\ A_27b)\ V0e)$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (11)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EFOLDL \\ A_27a\ A_27b \in (((A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \end{aligned} \quad (12)$$

Definition 15 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Elist_2Elist\ A_27a)\ (c_2Elist_2Elist\ A_27b))$

Let $c_2Elist_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EREVERSE\ A_27a \in ((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)}) \quad (13)$$

Definition 16 We define $c_2Efmmapal_2Efmmap$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0l \in (ty_2Elist_2Elist\ (ty_2Elist_2Elist\ A_27a))$

Let $c_2Elist_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EAPPEND\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Elist_2Elist\ A_27a)}) \quad (14)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (15)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (16)$$

Assume the following.

$$True \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27))))))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b). (\forall V1a \in \\ & A_27a. (\forall V2b \in A_27b. (\forall V3x \in A_27a. ((ap (ap (c_2Efinite_map_2EFAPPLY \\ & A_27a\ A_27b) (ap (ap (c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b) V0f) \\ & (ap (ap (c_2Epair_2E_2C\ A_27a\ A_27b) V1a) V2b))) V3x) = (ap (ap (ap \\ & (c_2Ebool_2ECOND A_27b) (ap (ap (c_2Emin_2E_3D\ A_27a) V3x) V1a)) \\ & V2b) (ap (ap (c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b) V0f) V3x))))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0fm \in (ty_2Efinite_map_2Efmap\ A_27a\ A_27b). (\forall V1kvl1 \in \\ & (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). (\forall V2kvl2 \in \\ & (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). ((ap (ap (\\ & c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b) V0fm) (ap (ap (c_2Elist_2EAPPEND \\ & (ty_2Epair_2Eprod\ A_27a\ A_27b) V1kvl1) V2kvl2)) = (ap (ap (c_2Efinite_map_2EFUPDATE_LIST \\ & A_27a\ A_27b) (ap (ap (c_2Efinite_map_2EFUPDATE_LIST\ A_27a\ A_27b) \\ & V0fm) V1kvl1)) V2kvl2)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0f \in ((A_27b^{A_27a})^{A_27b}). (\forall V1e \in A_27b. ((ap (\\ & ap (ap (c_2Elist_2EFOLDL\ A_27a\ A_27b) V0f) V1e) (c_2Elist_2ENIL \\ & A_27a)) = V1e))) \wedge (\forall V2f \in ((A_27b^{A_27a})^{A_27b}). (\forall V3e \in \\ & A_27b. (\forall V4x \in A_27a. (\forall V5l \in (ty_2Elist_2Elist\ A_27a). \\ & ((ap (ap (ap (c_2Elist_2EFOLDL\ A_27a\ A_27b) V2f) V3e) (ap (ap (c_2Elist_2ECONS \\ & A_27a) V4x) V5l)) = (ap (ap (ap (c_2Elist_2EFOLDL\ A_27a\ A_27b) V2f) \\ & (ap (ap V2f V3e) V4x)) V5l))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2EREVERSE\ A_27a) \\
& \quad (c_2Elist_2ENIL\ A_27a)) = (c_2Elist_2ENIL\ A_27a)) \wedge (\forall V0h \in \\
& \quad A_27a. (\forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2EREVERSE \\
& \quad A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Elist_2EAPPEND \\
& \quad A_27a)\ (ap\ (c_2Elist_2EREVERSE\ A_27a)\ V1t))\ (ap\ (ap\ (c_2Elist_2ECONS \\
& \quad A_27a)\ V0h)\ (c_2Elist_2ENIL\ A_27a))))))
\end{aligned} \tag{26}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1y \in A_27a. (\forall V2z \in A_27b. (\forall V3l \in \\
& \quad (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)). ((ap\ (ap\ (\\
& \quad c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b)\ (ap\ (c_2Efmapal_2Efmap \\
& \quad A_27a\ A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ A_27a \\
& \quad A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1y)\ V2z))\ V3l))))\ V0x) = \\
& \quad (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27b)\ (ap\ (ap\ (c_2Emin_2E_3D\ A_27a) \\
& \quad V0x)\ V1y))\ V2z)\ (ap\ (ap\ (c_2Efinite_map_2EFAPPLY\ A_27a\ A_27b) \\
& \quad (ap\ (c_2Efmapal_2Efmap\ A_27a\ A_27b)\ V3l))\ V0x))))))
\end{aligned}$$