

thm_2Efmmapal_2Efmmap__FDOM__rec (TMPz- ZDMNqmMd1yWbW89g7p5vhxmkp5suFoY)

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Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define `c_2Ebool_2EF` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let `ty_2Elist_2Elist` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (2)$$

Let `c_2Elist_2EREVERSE` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2EREVERSE A_27a \in ((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)}) \quad (3)$$

Let `ty_2Eone_2Eone` : ι be given. Assume the following.

$$nonempty ty_2Eone_2Eone \quad (4)$$

Definition 6 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define `c_2Eone_2Eone` to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 8 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Esum_2Esum A0 A1) \quad (5)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EABS_sum A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (6)$$

Definition 10 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_2Esum_2EABS_sum$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efinite_map_2Efmap A0 A1) \quad (7)$$

Let $c_2Efinite_map_2Efmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_ABS A_27a A_27b \in ((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a}}) \quad (8)$$

Definition 11 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap (c_2Efinite_map_2Efmap$

Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EFUPDATE A_27a A_27b \in (((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)})^{(ty_2Efinite_map_2Efmap A_27a A_27b)}) \quad (9)$$

Let $c_2Elist_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2EFOLDL A_27a A_27b \in (((A_27b)^{(ty_2Elist_2Elist A_27a)})^{A_27b})^{((A_27b)^{A_27a})^{A_27b}} \quad (10)$$

Definition 12 We define $c_2Efinite_map_2EFUPDATE_LIST$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap (c_2Elist_2EFOLDL$

Definition 13 We define $c_2Efmapal_2Efmap$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0l \in (ty_2Elist_2Elist (ty_2Epair$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_REP A_27a A_27b \in (((ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap A_27a A_27b)}) \quad (11)$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EISL A_27a A_27b \in (2^{(ty_2Esum_2Esum A_27a A_27b)}) \quad (12)$$

Definition 14 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map)$

Let $c_2Elist_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2EMAP \\ A_27a\ A_27b \in (((ty_2Elist_2Elist\ A_27b)^{(ty_2Elist_2Elist\ A_27a)})^{(A_27b)^{A_27a}}) \end{aligned} \quad (13)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (14)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (15)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (16)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (17)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2Eprod\ A_27a\ A_27b)\ (V0x\ V1y))$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (18)$$

Definition 16 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.(c_2Ebool_2E_21\ 2)\ V2t))\ V0t1))$

Definition 18 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (19)$$

Definition 19 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Ebool_2E_5C_2F\ V1s)\ V0x)$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad \forall V0l \in (ty_2Elist_2Elist (ty_2Epair_2Eprod A_27a A_27b)). \\ & \quad ((ap (c_2Efinite_map_2EFDOM A_27a A_27b) (ap (c_2Efmamal_2Efmamal A_27a A_27b) V0l)) = (ap (c_2Elist_2ELIST_TO_SET A_27a) (ap (\\ & \quad ap (c_2Elist_2EMAP (ty_2Epair_2Eprod A_27a A_27b) A_27a) (c_2Epair_2EFST A_27a A_27b)) V0l)))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \quad (\forall V0f \in (A_27b^{A_27a}).((ap (ap (c_2Elist_2EMAP A_27a A_27b) V0f) (c_2Elist_2ENIL A_27a)) = (c_2Elist_2ENIL A_27b))) \wedge (\forall V1f \in \\ & \quad (A_27b^{A_27a}).(\forall V2h \in A_27a.(\forall V3t \in (ty_2Elist_2Elist A_27a).((ap (ap (c_2Elist_2EMAP A_27a A_27b) V1f) (ap (ap (c_2Elist_2ECONS A_27a) V2h) V3t)) = (ap (ap (c_2Elist_2ECONS A_27b) (ap V1f V2h)) (ap (ap (c_2Elist_2EMAP A_27a A_27b) V1f) V3t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0h \in A_27b. (\forall V1t \in (ty_2Elist_2Elist\ A_27b). (\\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27a)\ (c_2Elist_2ENIL\ A_27a)) = \\ (c_2Epred_set_2EEMPTY\ A_27a)) \wedge ((ap\ (c_2Elist_2ELIST_TO_SET \\ A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c_2Epred_set_2EINSERT \\ A_27b)\ V0h)\ (ap\ (c_2Elist_2ELIST_TO_SET\ A_27b)\ V1t)))))) \\ (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (c_2Epair_2EFST\ A_27a \\ A_27b)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y)) = V0x))) \\ (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg (p\ (ap\ (ap \\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \\ (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s)))))) \\ (32) \end{aligned}$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (\\ c_2Efinite_map_2EFDOM\ A_27a\ A_27b)\ (ap\ (c_2Efmopal_2Efmopal\ A_27a \\ A_27b)\ (c_2Elist_2ENIL\ (ty_2Epair_2Eprod\ A_27a\ A_27b)))))) \Leftrightarrow \\ False)) \wedge (\forall V1x \in A_27a. (\forall V2w \in A_27a. (\forall V3z \in \\ A_27b. (\forall V4l \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a \\ A_27b)). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x)\ (ap\ (c_2Efinite_map_2EFDOM \\ A_27a\ A_27b)\ (ap\ (c_2Efmopal_2Efmopal\ A_27a\ A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS \\ (ty_2Epair_2Eprod\ A_27a\ A_27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a \\ A_27b)\ V2w)\ V3z))\ V4l)))))) \Leftrightarrow ((V1x = V2w) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V1x)\ (ap\ (c_2Efinite_map_2EFDOM\ A_27a\ A_27b)\ (ap\ (c_2Efmopal_2Efmopal \\ A_27a\ A_27b)\ V4l)))))))))) \\ \end{aligned}$$