thm_2Efmapal_2Emerge__out__ind (TMG5nSQq87a4L2rVEzc9eQjWmZaysJk8p1v)

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Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{1}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{2}$$

Let $ty_2Etoto_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etoto_2Etoto\ A0) \tag{3}$$

Let $c_2Efmapal_2Emerge : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow A_27a\ A_27b \in ((((ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b))^{(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_2} \tag{4}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{5}$$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \tag{6}$$

Let $c_2EbasicSize_2Eoption__size : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2EbasicSize_2Eoption__size\ A_27a \in ((ty_2Enum_2Enum^{(ty_2Eoption_2Eoption\ A_27a)})^{(ty_2Enum_2Enum^{A_27a})}) \tag{7}$$

Let $c_2Elist_2Elist__size : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2Elist__size\ A_27a \in ((ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)})^{(ty_2Enum_2Enum^{A_27a})}) \tag{8}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \tag{9}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \tag{10}$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \tag{11}$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{12}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^{A_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c_2Emin_2E_3D_3D_3E}\ V0t)\ \mathsf{c_2Ebool_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in 2$

Let $c_2Enum_2EREP__num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP__num \in (omega^{ty_2Enum_2Enum}) \tag{13}$$

Let $c_2Enum_2ESUC__REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC__REP \in (omega^{omega}) \tag{14}$$

Let $c_2Enum_2EABS__num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS__num \in (ty_2Enum_2Enum^{omega}) \tag{15}$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS__num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (\mathsf{c_2Emin_2E_40}$

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2En$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Let $c_2Enum_2EZERO__REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO__REP \in omega \tag{16}$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c_2Enum_2EABS__num\ c_2Enum_2EZERO__REP)$.

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 17 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2E}$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{17}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{18}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{19}$$

Definition 18 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 19 *We define* c_2Enumeral_2EiiSUC *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ \mathsf{c_2Enum_2ESUC}\ (ap$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{20}$$

Definition 20 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 21 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 22 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 23 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 24 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{21}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{22}$$

Let $c_2Esum_2EABS__sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS__sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{(((2^{A_27b})^{A_27a})^2)}) \tag{23}$$

Definition 25 *We define* c_2Esum_2EINL *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption__ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption__ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \tag{24}$$

Definition 26 *We define* c_2Eoption_2ESOME *to be* $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption__$

Definition 27 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c_2Emin_2E_40}\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2$

Definition 28 *We define* c_2Esum_2EINR *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS$

Definition 29 *We define* c_2Eoption_2ENONE *to be* $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption__ABS\ A_27a)\ ($

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{25}$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{26}$$

Let $c_2Epair_2EABS__prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS__prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})}) \tag{27}$$

Definition 30 *We define* c_2Epair_2E_2C *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Definition 31 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27}$

Definition 32 *We define* c_2Erelation_2Einv__image *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27b})^{A_27b}).\lambda V$

Definition 33 *We define* c_2Erelation_2EWF *to be* $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap$ (c_2Ebool_2E_21

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ \mathsf{c_2Enum_2E0}) = V0m)) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ \mathsf{c_2Enum_2E0})\ V0m) = V0m)\wedge(((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ \mathsf{c_2Enum_2E0}) = V0m)\wedge(((ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Enum_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n)))\wedge((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ \mathsf{c_2Enum_2ESUC}\ V1n)) = (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n)))))))) \tag{29}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V0m)))) \tag{30}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V0m)))) \tag{31}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V2p)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ V2p))))) \tag{32}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V0m)\ V1n))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Enum_2ESUC}\ V0m))\ V1n))))) \tag{33}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ \mathsf{c_2Enum_2E0})\ V0n))) \tag{34}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V1n)\ V0m)))))) \tag{35}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap\ (ap\ c_2Earithmetic_2E_2A\ \mathsf{c_2Enum_2E0})\ V0m) = \mathsf{c_2Enum_2E0})\wedge(((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ \mathsf{c_2Enum_2E0}) = \mathsf{c_2Enum_2E0})\wedge(((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))\ V0m) = V0m)\wedge(((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO}))) = V0m)\wedge((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ \mathsf{c_2Enum_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))\ V1n))\wedge((ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ (ap\ \mathsf{c_2Enum_2ESUC}\ V1n)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V0m)\ V1n))))))))) \tag{36}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.(((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V1n)\ V2p)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V0m)\ V2p)))))) \tag{37}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V1n)\ V2p)))))) \tag{38}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap\ \mathsf{c_2Enum_2ESUC}\ V0n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))\ V0n))) \tag{39}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow((\forall V0f \in (ty_2Enum_2Enum^{A_27a}).((ap\ (ap\ (c_2EbasicSize_2Eoption__size\ A_27a)\ V0f)\ (\mathsf{c_2Eoption_2ENONE}\ A_27a)) = \mathsf{c_2Enum_2E0}))\wedge(\forall V1f \in (ty_2Enum_2Enum^{A_27a}).(\forall V2x \in A_27a.((ap\ (ap\ (c_2EbasicSize_2Eoption__size\ A_27a)\ V1f)\ (ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)\ V2x)) = (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ V1f\ V2x)))))) \tag{40}$$

Assume the following.

$$True \tag{41}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{42}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{46}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{47}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{49}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\wedge(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\vee(\neg(p\ V1B))))\wedge((\neg((p\ V0A)\vee(p\ V1B)))\Leftrightarrow((\neg(p\ V0A))\wedge(\neg(p\ V1B))))))) \tag{51}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t)\Rightarrow False)\Leftrightarrow((p\ V0t)\Leftrightarrow False))) \tag{52}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{53}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Leftrightarrow(p\ V1t2))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\vee((\neg(p\ V0t1))\wedge(\neg(p\ V1t2))))))) \tag{54}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow((\forall V0f \in (ty_2Enum_2Enum^{A_27a}).((ap\ (ap\ (c_2Elist_2Elist__size\ A_27a)\ V0f)\ (c_2Elist_2ENIL\ A_27a)) = \mathsf{c_2Enum_2E0}))\wedge(\forall V1f \in (ty_2Enum_2Enum^{A_27a}).(\forall V2a0 \in A_27a.(\forall V3a1 \in (ty_2Elist_2Elist\ A_27a).((ap\ (ap\ (c_2Elist_2Elist__size\ A_27a)\ V1f)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0)\ V3a1)) = (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ V1f\ V2a0))\ (ap\ (ap\ (c_2Elist_2Elist__size\ A_27a)\ V1f)\ V3a1)))))))) \tag{55}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow(\forall V0l \in (ty_2Elist_2Elist\ A_27a).((V0l = (c_2Elist_2ENIL\ A_27a))\vee(\exists V1h \in A_27a.(\exists V2t \in (ty_2Elist_2Elist\ A_27a).(V0l = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V1h)\ V2t)))))) \tag{56}$$

Assume the following.

$$((\forall V0n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ \mathsf{c_2Enum_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ \mathsf{c_2Enum_2E0}) = V1n))\wedge((\forall V2n \in ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V3m)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Enumeral_2EiZ}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2A\ \mathsf{c_2Enum_2E0})\ V4n) = \mathsf{c_2Enum_2E0}))\wedge((\forall V5n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2A\ V5n)\ \mathsf{c_2Enum_2E0}) = \mathsf{c_2Enum_2E0}))\wedge((\forall V6n \in ty_2Enum_2Enum.(\forall V7m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2A\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V7m)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ (ap\ c_2Earithmetic_2E_2A\ V6n)\ V7m)))))\wedge((\forall V8n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2D\ \mathsf{c_2Enum_2E0})\ V8n) = \mathsf{c_2Enum_2E0}))\wedge((\forall V9n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2D\ V9n)\ \mathsf{c_2Enum_2E0}) = V9n))\wedge((\forall V10n \in ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V11m)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ (ap\ c_2Earithmetic_2E_2D\ V10n)\ V11m)))))\wedge((\forall V12n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2EEXP\ \mathsf{c_2Enum_2E0})\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V12n))) = \mathsf{c_2Enum_2E0}))\wedge((\forall V13n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2EEXP\ \mathsf{c_2Enum_2E0})\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V13n))) = \mathsf{c_2Enum_2E0}))\wedge((\forall V14n \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2EEXP\ V14n)\ \mathsf{c_2Enum_2E0}) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO}))))\wedge((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum.((ap\ (ap\ c_2Earithmetic_2EEXP\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V15n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ (ap\ c_2Earithmetic_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c_2Enum_2ESUC}\ \mathsf{c_2Enum_2E0}) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ \mathsf{c_2Earithmetic_2EZERO})))\wedge((\forall V17n \in ty_2Enum_2Enum.((ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Enum_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c_2Eprim__rec_2EPRE}\ \mathsf{c_2Enum_2E0}) = \mathsf{c_2Enum_2E0})\wedge((\forall V18n \in ty_2Enum_2Enum.((ap\ \mathsf{c_2Eprim__rec_2EPRE}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Eprim__rec_2EPRE}\ V18n))))\wedge((\forall V19n \in ty_2Enum_2Enum.(((ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V19n) = \mathsf{c_2Enum_2E0})\Leftrightarrow(V19n = \mathsf{c_2Earithmetic_2EZERO})))\wedge((\forall V20n \in ty_2Enum_2Enum.((\mathsf{c_2Enum_2E0} = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V20n))\Leftrightarrow(V20n = \mathsf{c_2Earithmetic_2EZERO})))\wedge((\forall V21n \in ty_2Enum_2Enum.(\forall V22m \in ty_2Enum_2Enum.(((ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V21n) = (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge((\forall V23n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V23n)\ \mathsf{c_2Enum_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ \mathsf{c_2Enum_2E0})\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ \mathsf{c_2Earithmetic_2EZERO})\ V24n))))\wedge((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V25n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V25n)\ V26m)))))\wedge((\forall V27n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3E}\ \mathsf{c_2Enum_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3E}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V28n))\ \mathsf{c_2Enum_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ \mathsf{c_2Earithmetic_2EZERO})\ V28n))))\wedge((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3E}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V29n))\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V30m)\ V29n)))))\wedge((\forall V31n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ \mathsf{c_2Enum_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(((ap\ \mathsf{c_2Enumeral_2EiZ}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ \mathsf{c_2Earithmetic_2EZERO})\ V0n)) = V0n) \wedge (((ap\ \mathsf{c_2Enumeral_2EiZ}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ \mathsf{c_2Earithmetic_2EZERO})) = V0n) \wedge (((ap\ \mathsf{c_2Enumeral_2EiZ}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ (ap\ \mathsf{c_2Enumeral_2EiZ}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (((ap\ \mathsf{c_2Enumeral_2EiZ}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (((ap\ \mathsf{c_2Enumeral_2EiZ}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (((ap\ \mathsf{c_2Enumeral_2EiZ}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (((ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ \mathsf{c_2Earithmetic_2EZERO})\ V0n)) = (ap\ \mathsf{c_2Enum_2ESUC}\ V0n)) \wedge (((ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ \mathsf{c_2Earithmetic_2EZERO})) = (ap\ \mathsf{c_2Enum_2ESUC}\ V0n)) \wedge (((ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (((ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (((ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (((ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ (ap\ \mathsf{c_2Enumeral_2EiiSUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (((ap\ \mathsf{c_2Enumeral_2EiiSUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ \mathsf{c_2Earithmetic_2EZERO})\ V0n)) = (ap\ \mathsf{c_2Enumeral_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c_2Enumeral_2EiiSUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ \mathsf{c_2Earithmetic_2EZERO})) = (ap\ \mathsf{c_2Enumeral_2EiiSUC}\ V0n)) \wedge (((ap\ \mathsf{c_2Enumeral_2EiiSUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ (ap\ \mathsf{c_2Enum_2ESUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge (((ap\ \mathsf{c_2Enumeral_2EiiSUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ (ap\ \mathsf{c_2Enumeral_2EiiSUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))) \wedge ((ap\ \mathsf{c_2Enumeral_2EiiSUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m))) = (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ (ap\ \mathsf{c_2Enumeral_2EiiSUC}\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0n)\ V1m)))))))))))))))))))))))) \tag{58}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ \mathsf{c_2Earithmetic_2EZERO})\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ \mathsf{c_2Earithmetic_2EZERO})\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n)))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V0n)\ \mathsf{c_2Earithmetic_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Eprim__rec_2E_3C}\ V0n)\ V1m)))))))))))) \tag{59}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ \mathsf{c_2Earithmetic_2EZERO})\ V0n))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ \mathsf{c_2Earithmetic_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ \mathsf{c_2Earithmetic_2EZERO}))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m)))\Leftrightarrow(\neg(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V1m)\ V0n))))\wedge((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ V0n)\ V1m))))))))))) \tag{60}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow(\forall V0opt \in (ty_2Eoption_2Eoption\ A_27a).((V0opt = (\mathsf{c_2Eoption_2ENONE}\ A_27a))\vee(\exists V1x \in A_27a.(V0opt = (ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)\ V1x))))) \tag{61}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a\Rightarrow\forall A_27b.nonempty\ A_27b\Rightarrow(\forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).(\exists V1q \in A_27a.(\exists V2r \in A_27b.(V0x = (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27a\ A_27b)\ V1q)\ V2r))))) \tag{62}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).((ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27a\ A_27b)\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0x))\ (ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ V0x)) = V0x)) \tag{63}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.((ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27a\ A_27b)\ V0x)\ V1y)) = V1y))) \tag{64}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}).(\forall V1x \in A_27a.(\forall V2y \in A_27b.((ap\ (ap\ (\mathsf{c_2Epair_2EUNCURRY}\ A_27a\ A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27a\ A_27b)\ V1x)\ V2y)) = (ap\ (ap\ V0f\ V1x)\ V2y))))) \tag{65}$$

Assume the following.

$$(p\ (ap\ (\mathsf{c_2Erelation_2EWF}\ ty_2Enum_2Enum)\ \mathsf{c_2Eprim__rec_2E_3C})) \tag{66}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). ((p\ (ap\ (\mathsf{c_2Erelation_2EWF}\ A_27a)\ V0R)) \Rightarrow (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a.((\forall V3y \in A_27a.((p\ (ap\ (ap\ V0R\ V3y)\ V2x)) \Rightarrow (p\ (ap\ V1P\ V3y)))) \Rightarrow (p\ (ap\ V1P\ V2x)))) \Rightarrow (\forall V4x \in A_27a.(p\ (ap\ V1P\ V4x))))))) \tag{67}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0R \in ((2^{A_27b})^{A_27b}).(\forall V1f \in (A_27b^{A_27a}).((p\ (ap\ (\mathsf{c_2Erelation_2EWF}\ A_27b)\ V0R)) \Rightarrow (p\ (ap\ (\mathsf{c_2Erelation_2EWF}\ A_27a)\ (ap\ (ap\ (\mathsf{c_2Erelation_2Einv__image}\ A_27a\ A_27b)\ V0R)\ V1f)))))) \tag{68}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{77}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27$$
$$\forall V0P \in (((2^{(ty_2Elist_2Elist\ (ty_2Eoption_2Eoption\ (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b))))})^{(ty_2Elist_2}$$
$$(((\forall V1cmp \in (ty_2Etoto_2Etoto\ A_27a).(\forall V2l \in (ty_2Eli$$
$$(ty_2Epair_2Eprod\ A_27a\ A_27b)).(p\ (ap\ (ap\ (ap\ V0P\ V1$$
$$c_2Elist_2ENIL\ (ty_2Eoption_2Eoption\ (ty_2Elist_2Elist\ (ty_$$
$$A_27a\ A_27b))))))))\wedge((\forall V3cmp \in (ty_2Etoto_2Etoto\ A$$
$$(\forall V4l \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A$$
$$(\forall V5lol \in (ty_2Elist_2Elist\ (ty_2Eoption_2Eoption\ (ty_2E$$
$$(ty_2Epair_2Eprod\ A_27a\ A_27b)))).((p\ (ap\ (ap\ (ap\ V0P\ V$$
$$V5lol))\Rightarrow(p\ (ap\ (ap\ (ap\ V0P\ V3cmp)\ V4l)\ (ap\ (ap\ (c_2Elis$$
$$ty_2Eoption_2Eoption\ (ty_2Elist_2Elist\ (ty_2Epair_2Ep$$
$$A_27b)))))\ (\mathsf{c_2Eoption_2ENONE}\ (ty_2Elist_2Elist\ (ty_2Ep$$
$$A_27a\ A_27b))))\ V5lol)))))))\wedge(\forall V6cmp \in (ty_2Etoto_2$$
$$A_27a).(\forall V7l \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod$$
$$A_27b)).(\forall V8m \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod$$
$$A_27b)).(\forall V9lol \in (ty_2Elist_2Elist\ (ty_2Eoption_2E$$
$$(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)))).$$
$$(ap\ (ap\ V0P\ V6cmp)\ (ap\ (ap\ (ap\ (c_2Efmapal_2Emerge\ A_27a$$
$$V7l)\ V8m))\ V9lol))\Rightarrow(p\ (ap\ (ap\ (ap\ V0P\ V6cmp)\ V7l)\ (ap\ (ap\ ($$
$$(ty_2Eoption_2Eoption\ (ty_2Elist_2Elist\ (ty_2Epair_2$$
$$A_27a\ A_27b))))\ (ap\ (\mathsf{c_2Eoption_2ESOME}\ (ty_2Elist_2Elist\ (ty$$
$$A_27a\ A_27b)))\ V8m))\ V9lol)))))))))\Rightarrow(\forall V10v \in (ty_2Et$$
$$A_27a).(\forall V11v1 \in (ty_2Elist_2Elist\ (ty_2Epair_2E$$
$$A_27a\ A_27b)).(\forall V12v2 \in (ty_2Elist_2Elist\ (ty_2Eoption$$
$$(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ A_27a\ A_27b)))).$$
$$ap\ (ap\ V0P\ V10v)\ V11v1)\ V12v2)))))))$$