thm_2Efmapal_2Eo__f__fmap (TMSuPco7QuiBvQBMExaVVE72TCq8dSR7goe)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $ty\_2Efinite\_\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_\_map\_2Efmap\ A0\ A1) \tag{3}$$

Let $c\_2Efinite\_\_map\_2Efmap\_\_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)}) \quad (4)$$

Let $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EOUTL\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \quad (5)$$

Definition 10 *We define* c_2Efinite__map_2EFAPPLY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_ma$

Let $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EISL\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \quad (6)$$

Definition 11 *We define* c_2Efinite__map_2EFDOM *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_map$

Let $c\_2Efinite\_\_map\_2Eo\_\_f : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow c\_2Efinite\_\_map\_2Eo\_\_f\ A\_27a\ A\_27b\ A\_27c \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27c)^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)})^{(A\_27c^{A\_27b})}) \quad (7)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (8)$$

Let $c\_2Efinite\_\_map\_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2EFUPDAT\ A\_27a\ A\_27b \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})^{(ty\_2Efinite\_\_map} \quad (9)$$

Definition 12 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (10)$$

Definition 13 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Efinite\_\_map\_2Efmap\_\_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_ABS\ A\_27a\ A\_27b \in ((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)^{((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})}) \quad (11)$$

Definition 14 *We define* c_2Efinite__map_2EFEMPTY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Efinite\_\_map\_2E$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{12}$$

Let $c\_2Elist\_2EFOLDL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27b})^{((A\_27b^{A\_27a})^{A\_27b})}) \tag{13}$$

Definition 15 *We define* c_2Efinite__map_2EFUPDATE__LIST *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Elist\_2E$

Let $c\_2Elist\_2EREVERSE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EREVERSE\ A\_27a \in ((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{14}$$

Definition 16 *We define* c_2Efmapal_2Efmap *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0l \in (ty\_2Elist\_2Elist\ (ty\_2Ep$

Let $c\_2Efmapal\_2EAP\_\_SND : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow c\_2Efmapal\_2EAP\_\_SND\ A\_27a\ A\_27b\ A\_27c \in (((ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})^{(A\_27c^{A\_27b})}) \tag{15}$$

Let $c\_2Elist\_2EMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2EMAP\ A\_27a\ A\_27b \in (((ty\_2Elist\_2Elist\ A\_27b)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{16}$$

Let $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{17}$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{18}$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{19}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{20}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{21}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{22}$$

Definition 17 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 18 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Definition 19 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 20 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{23}$$

Definition 21 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Assume the following.

$$True \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \land (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \land (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1a \in A\_27a.(\forall V2b \in A\_27b.((ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1a)\ V2b))) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1a)\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ V0f)))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1a \in A\_27a.(\forall V2b \in A\_27b.(\forall V3x \in A\_27a.((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFAPPLY}\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1a)\ V2b)))\ V3x) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ V3x)\ V1a))\ V2b)\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFAPPLY}\ A\_27a\ A\_27b)\ V0f)\ V3x))))))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1g \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).((V0f = V1g) \Leftrightarrow (((ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ V0f) = (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ V1g)) \wedge (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ V0f))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFAPPLY}\ A\_27a\ A\_27b)\ V0f)\ V2x) = (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFAPPLY}\ A\_27a\ A\_27b)\ V1g)\ V2x)))))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27b}).(\forall V1g \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).((ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27c)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2Eo\_\_f\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1g)) = (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27a\ A\_27b)\ V1g)))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27c}).((ap\ (ap\ (c\_2Efinite\_\_map\_2Eo\_\_f\ A\_27a\ A\_27c\ A\_27b)\ V0f)\ (\mathsf{c\_2Efinite\_\_map\_2EFEMPTY}\ A\_27a\ A\_27c)) = (\mathsf{c\_2Efinite\_\_map\_2EFEMPTY}\ A\_27a\ A\_27b))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27c}).(\forall V1fm \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27c).(\forall V2k \in A\_27a.(\forall V3v \in A\_27c.((ap\ (ap\ (c\_2Efinite\_\_map\_2Eo\_\_f\ A\_27a\ A\_27c\ A\_27b)\ V0f)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27a\ A\_27c)\ V1fm)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27c)\ V2k)\ V3v))) = (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2Eo\_\_f\ A\_27a\ A\_27c\ A\_27b)\ V0f)\ V1fm))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2k)\ (ap\ V0f\ V3v))))))))\tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ V0f)\ (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))) = V0f) \wedge (\forall V1h \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\forall V2t \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ V0f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V1h)\ V2t)) = (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Efinite\_\_map\_2EFUPDATE\ A\_27a\ A\_27b)\ V0f)\ V1h))\ V2t))))))\tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0fm \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b).(\forall V1kvl1 \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).(\forall V2kvl2 \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).((ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ V0fm)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V1kvl1)\ V2kvl2)) = (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFUPDATE\_\_LIST}\ A\_27a\ A\_27b)\ V0fm)\ V1kvl1))\ V2kvl2)))))\tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27b}).(\forall V1a \in A\_27a.(\forall V2b \in A\_27b.((ap\ (ap\ (c\_2Efmapal\_2EAP\_\_SND\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1a)\ V2b)) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27c)\ V1a)\ (ap\ V0f\ V2b))))))\tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V0f)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27b)))\wedge(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V1f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V3t)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ (ap\ V1f\ V2h))\ (ap\ (ap\ (c\_2Elist\_2EMAP\ A\_27a\ A\_27b)\ V1f)\ V3t))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V1e \in A\_27b.((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (c\_2Elist\_2ENIL\ A\_27a)) = V1e)))\wedge(\forall V2f \in ((A\_27b^{A\_27a})^{A\_27b}).(\forall V3e \in A\_27b.(\forall V4x \in A\_27a.(\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4x)\ V5l)) = (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDL\ A\_27a\ A\_27b)\ V2f)\ (ap\ (ap\ V2f\ V3e)\ V4x))\ V5l))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a)))\wedge(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t))\Rightarrow(\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))))\Rightarrow(\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) = (c\_2Elist\_2ENIL\ A\_27a))\wedge(\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (c\_2Elist\_2EREVERSE\ A\_27a)\ V1t))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ (c\_2Elist\_2ENIL\ A\_27a))))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0x))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V0x)) = V0x)) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y)) = (ap\ (ap\ V0f\ V1x)\ V2y))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\forall V2y \in A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ 2)\ (\lambda V3x \in A\_27a.(\lambda V4y \in A\_27b.(ap\ (ap\ V0P\ V3x)\ V4y)))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = V1y) \vee (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s))))))) \tag{45}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27b}).(\forall V1l \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).((ap\ (ap\ (c\_2Efinite\_\_map\_2Eo\_\_f\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (\mathsf{c\_2Efmapal\_2Efmap}\ A\_27a\ A\_27b)\ V1l)) = (ap\ (\mathsf{c\_2Efmapal\_2Efmap}\ A\_27a\ A\_27c)\ (ap\ (ap\ (c\_2Elist\_2EMAP\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))\ (ap\ (c\_2Efmapal\_2EAP\_\_SND\ A\_27a\ A\_27b\ A\_27c)\ V0f))\ V1l)))))$$