

thm_2Efmaptree_2Eapplicable__paths__FINITE (TMap3GME65jwC16K4UazT1D5W58GUJJxmGM)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V 0x \in 2.V 0x)) (\lambda V 1x \in 2.V 1x)$

Definition 3 We define `c_2Ebool_2EBOUNDED` to be $(\lambda V 0v \in 2. \text{c_2Ebool_2ET})$.

Let $\text{ty_2Efmaptree_2Efmaptree} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Efmaptree_2Efmaptree } A0 \ A1) \quad (1)$$

Let $\text{ty_2Efinite_map_2Efmap} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Efinite_map_2Efmap } A0 \ A1) \quad (2)$$

Let $\text{c_2Efmaptree_2Emap} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Efmaptree_2Emap } A_27a \ A_27b \in ((\text{ty_2Efinite_map_2Efmap } A_27a \ (\text{ty_2Efmaptree_2Efmaptree } A_27a \ A_27b))^{(\text{ty_2Efmaptree_2Efmaptree } A_27a \ A_27b)}) \quad (3)$$

Let $\text{ty_2Eoption_2Eoption} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eoption_2Eoption } A0) \quad (4)$$

Let $\text{ty_2Elist_2Elist} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Elist_2Elist } A0) \quad (5)$$

Let $\text{c_2Efmaptree_2Eapply_path} : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \text{c_2Efmaptree_2Eapply_path } A_27a \ A_27b \in (((\text{ty_2Eoption_2Eoption } (\text{ty_2Efmaptree_2Efmaptree } A_27a \ A_27b))^{(\text{ty_2Efmaptree_2Efmaptree } A_27a \ A_27b)})^{(\text{ty_2Elist_2Elist } A_27a)}) \quad (6)$$

Let $c_2Efmptree_2EtoF : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27key.nonempty\ A_27key \Rightarrow \forall A_27value.nonempty \\ & A_27value \Rightarrow c_2Efmptree_2EtoF\ A_27key\ A_27value \in (((ty_2Eoption_2Eoption \\ & A_27value)^{(ty_2Elist_2Elist\ A_27key)})^{(ty_2Efmptree_2Efmptree\ A_27key\ A_27value)}) \end{aligned} \quad (7)$$

Let $c_2Efinite_map_2Eo_f : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow c_2Efinite_map_2Eo_f\ A_27a\ A_27b\ A_27c \in (((\\ & ty_2Efinite_map_2Efmap\ A_27a\ A_27c)^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)})^{(A_27c^{A_27b})}) \end{aligned} \quad (8)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (9)$$

Definition 4 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p\ (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 5 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone$

Definition 6 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}$

Definition 7 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 8 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 9 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E2F$

Definition 10 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum \\ & A0\ A1) \end{aligned} \quad (10)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ & A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (11)$$

Definition 11 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ & ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (12)$$

Definition 12 We define $c_Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap (c_Eoption_2Eoption_ABS A_27a) (c_Eoption_2ENONE A_27a))$.
 Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_REP \\ & A_27a A_27b \in (((ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap A_27a A_27b)}) \end{aligned} \quad (13)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EOUTL \\ & A_27a A_27b \in (A_27a^{(ty_2Esum_2Esum A_27a A_27b)}) \end{aligned} \quad (14)$$

Definition 13 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map A_27a A_27b). (ap V0f (c_2Efinite_map_2EFAPPLY A_27a A_27b V0f))$.
 Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EISL \\ & A_27a A_27b \in (2^{(ty_2Esum_2Esum A_27a A_27b)}) \end{aligned} \quad (15)$$

Definition 14 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (ty_2Efinite_map A_27a A_27b). (ap V0f (c_2Efinite_map_2EFDOM A_27a A_27b V0f))$.

Definition 15 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$.

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a. (ap V2t2 (c_2Ebool_2ECOND A_27a V1t1 V2t2)))))$.

Definition 17 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a. (ap (c_2Esum_2EABS A_27a A_27b) V0e)$.

Definition 18 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS A_27a) V0x)$.

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Elist_2Elist_CASE \\ & A_27a A_27b \in (((A_27b^{(A_27b^{(ty_2Elist_2Elist A_27a A_27b)})^{A_27a}})^{A_27b})^{(ty_2Elist_2Elist A_27a A_27b)}) \end{aligned} \quad (16)$$

Definition 19 We define $c_2Efmaptree_2Econstruct$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in A_27a. \lambda V1kfm \in (ty_2Efmaptree A_27a A_27b). (ap V1kfm (c_2Efmaptree_2Econstruct A_27a A_27b V0a V1kfm))$.

Let $c_2Efmaptree_2EfromF : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27key.nonempty A_27key \Rightarrow \forall A_27value.nonempty A_27value \Rightarrow c_2Efmaptree_2EfromF A_27key A_27value \in ((ty_2Efmaptree_2Efmaptree \\ & A_27key A_27value)^{(ty_2Eoption_2Eoption A_27value)^{(ty_2Elist_2Elist A_27key)}}) \end{aligned} \quad (17)$$

Definition 20 We define $c_2Efmaptree_2EFTNode$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0i \in A_27b. \lambda V1kfm \in (ty_2Efmaptree A_27a A_27b). (ap V1kfm (c_2Efmaptree_2EFTNode A_27a A_27b V0i V1kfm))$.

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)(ty_2Elist_2Elist\ A_27a))A_27a) \quad (18)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (19)$$

Definition 21 We define $c_2Emarker_2EAbbrev$ to be $\lambda V0x \in 2.V0x$.

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_SOME\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (20)$$

Let $c_2Eoption_2EIS_NONE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_NONE\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (21)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (22)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (23)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (24)$$

Definition 22 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (25)$$

Definition 23 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{35}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t1 \in A_27a.(\forall V1t2 \in \\
& A_27a.(((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\
& V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \\
& V0t1) \ V1t2) = V1t2))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in \\
& 2.((\exists V2x \in A_27a.((p \ (ap \ V0P \ V2x)) \wedge (p \ V1Q))) \Leftrightarrow ((\exists V3x \in \\
& A_27a.(p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\
& 2^{A_27a}).((\exists V2x \in A_27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \\
& V0P) \wedge (\exists V3x \in A_27a.(p \ (ap \ V1Q \ V3x))))))
\end{aligned} \tag{41}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Leftrightarrow (p \ V1t2)) \Leftrightarrow (((p \ V0t1) \Rightarrow (p \ V1t2)) \wedge ((p \ V1t2) \Rightarrow (p \ V0t1)))))) \quad (43)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{27} \in 2.(\forall V2y \in 2.(\forall V3y_{27} \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_{27})) \wedge ((p \ V1x_{27}) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_{27})))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_{27}) \Rightarrow (p \ V3y_{27})))))) \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ (\forall V2x \in A_27a.(\forall V3x_{27} \in A_27a.(\forall V4y \in A_27a. \\ (\forall V5y_{27} \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge ((p \ V1Q) \Rightarrow (V2x = V3x_{27})) \wedge \\ ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_{27})))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \\ V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_{27} \\ V5y_{27})))))))))) \end{aligned} \quad (45)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0a \in A_27a.(\exists V1x \in A_27a.(V1x = V0a))) \quad (46)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (ap \ V0P \ V1a)))))) \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow ((\forall V0t1 \in A_27a.(\forall V1t2 \in \\ A_27a.((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2ET) \ V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a.(\forall V3t2 \in A_27a.((ap \\ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ c_2Ebool_2EF) \ V2t1) \ V3t2) = V3t2)))))) \end{aligned} \quad (48)$$

Assume the following.

$$(\forall V0v \in 2.((p \ (ap \ c_2Ebool_2EBOUNDED \ V0v)) \Leftrightarrow True)) \quad (49)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ \forall V0fm \in (ty_2Efinite_map_2Efmap \ A_27a \ A_27b).(p \ (ap \ (c_2Epred_set_2EFINITE \\ A_27a) \ (ap \ (c_2Efinite_map_2EFDOM \ A_27a \ A_27b) \ V0fm)))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0i \in A.27a. (\forall V1fm \in (ty_2Efinite_map_2Efm\ A.27b \\ & \quad (ty_2Efmptree_2Efmptree\ A.27b\ A.27a)). (ap\ (c_2Efmptree_2Emap \\ & \quad A.27b\ A.27a)\ (ap\ (ap\ (c_2Efmptree_2EFTNode\ A.27b\ A.27a)\ V0i)\ V1fm)) = \\ & \quad V1fm))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad (\forall V0ft \in (ty_2Efmptree_2Efmptree\ A.27a\ A.27b)). (ap\ (\\ & \quad ap\ (c_2Efmptree_2Eapply_path\ A.27a\ A.27b)\ (c_2Elist_2ENIL \\ & \quad A.27a))\ V0ft) = (ap\ (c_2Eoption_2ESOME\ (ty_2Efmptree_2Efmptree \\ & \quad A.27a\ A.27b))\ V0ft))) \wedge (\forall V1h \in A.27a. (\forall V2t \in (ty_2Elist_2Elist \\ & \quad A.27a). (\forall V3ft \in (ty_2Efmptree_2Efmptree\ A.27a\ A.27b). \\ & \quad ((ap\ (ap\ (c_2Efmptree_2Eapply_path\ A.27a\ A.27b)\ (ap\ (ap\ (c_2Elist_2ECONS \\ & \quad A.27a)\ V1h)\ V2t))\ V3ft) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\ & \quad (ty_2Efmptree_2Efmptree\ A.27a\ A.27b)))\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A.27a)\ V1h)\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ (ty_2Efmptree_2Efmptree \\ & \quad A.27a\ A.27b))\ (ap\ (c_2Efmptree_2Emap\ A.27a\ A.27b)\ V3ft))))\ (ap \\ & \quad (ap\ (c_2Efmptree_2Eapply_path\ A.27a\ A.27b)\ V2t)\ (ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\ & \quad A.27a\ (ty_2Efmptree_2Efmptree\ A.27a\ A.27b))\ (ap\ (c_2Efmptree_2Emap \\ & \quad A.27a\ A.27b)\ V3ft))\ V1h)))\ (c_2Eoption_2ENONE\ (ty_2Efmptree_2Efmptree \\ & \quad A.27a\ A.27b))))))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0P \in (2^{(ty_2Efmptree_2Efmptree\ A.27a\ A.27b)}). ((\forall V1a \in \\ & \quad A.27b. (\forall V2fm \in (ty_2Efinite_map_2Efm\ A.27a\ (ty_2Efmptree_2Efmptree \\ & \quad A.27a\ A.27b)). ((\forall V3k \in A.27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad A.27a)\ V3k)\ (ap\ (c_2Efinite_map_2EFDOM\ A.27a\ (ty_2Efmptree_2Efmptree \\ & \quad A.27a\ A.27b))\ V2fm))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (c_2Efinite_map_2EFAPPLY \\ & \quad A.27a\ (ty_2Efmptree_2Efmptree\ A.27a\ A.27b))\ V2fm)\ V3k)))) \Rightarrow \\ & \quad (p\ (ap\ V0P\ (ap\ (ap\ (c_2Efmptree_2EFTNode\ A.27a\ A.27b)\ V1a)\ V2fm)))))) \Rightarrow \\ & \quad (\forall V4ft \in (ty_2Efmptree_2Efmptree\ A.27a\ A.27b). (p\ (ap \\ & \quad V0P\ V4ft)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\ & \quad A.27a). ((V0l = (c_2Elist_2ENIL\ A.27a)) \vee (\exists V1h \in A.27a. (\\ & \quad \exists V2t \in (ty_2Elist_2Elist\ A.27a). (V0l = (ap\ (ap\ (c_2Elist_2ECONS \\ & \quad A.27a)\ V1h)\ V2t)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0a0 \in A_27a. (\forall V1a1 \in \\ (ty_2Elist_2Elist\ A_27a). (\forall V2a0_27 \in A_27a. (\forall V3a1_27 \in \\ (ty_2Elist_2Elist\ A_27a). (((ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0a0) \\ V1a1) = (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0_27)\ V3a1_27)) \Leftrightarrow ((V0a0 = \\ V2a0_27) \wedge (V1a1 = V3a1_27))))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0a1 \in (ty_2Elist_2Elist \\ A_27a). (\forall V1a0 \in A_27a. (\neg((c_2Elist_2ENIL\ A_27a) = (ap\ (\\ ap\ (c_2Elist_2ECONS\ A_27a)\ V1a0)\ V0a1)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ A_27a. (((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1X \in (\\ ty_2Eoption_2Eoption\ A_27a). (\forall V2x \in A_27a. (((ap\ (ap\ (\\ ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ V1X)\ (\\ c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow ((p\ V0P) \Rightarrow \\ (p\ (ap\ (c_2Eoption_2EIS_NONE\ A_27a)\ V1X)))) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ (c_2Eoption_2ENONE\ A_27a)) \\ V1X) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow ((p\ (ap\ (c_2Eoption_2EIS_SOME \\ A_27a)\ V1X)) \Rightarrow (p\ V0P))) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\ A_27a))\ V0P)\ V1X)\ (c_2Eoption_2ENONE\ A_27a)) = (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V2x)) \Leftrightarrow ((p\ V0P) \wedge (V1X = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2x)))) \wedge \\ (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a)) \\ V0P)\ (c_2Eoption_2ENONE\ A_27a))\ V1X) = (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V2x)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1X = (ap\ (c_2Eoption_2ESOME\ A_27a) \\ V2x))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ A_27b)\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V0x))\ (ap\ (c_2Epair_2ESND \\ & A_27a\ A_27b)\ V0x)) = V0x)) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0f \in ((A_27c^{A_27b})^{A_27a}).(\forall V1x \in \\ & A_27a.(\forall V2y \in A_27b.((ap\ (ap\ (c_2Epair_2EUNCURRY\ A_27a \\ & A_27b\ A_27c)\ V0f)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V1x)\ V2y))) = \\ & (ap\ (ap\ V0f\ V1x)\ V2y)))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ & (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}).(\forall V1v \in \\ & A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b.((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\neg(p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ & (2^{A_27a}).(\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V2x)\ (ap\ (ap\ (c_2Epred_set_2EUNION\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (65) \\ & (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V0s)) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V1t)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.(\forall V2s \in (2^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\ & V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s)))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (p\ (ap\ (c_2Epred_set_2ESING \\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY \\ A_27a)))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s))))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EUNION \\ A_27a)\ V0s)\ V1t))) \Leftrightarrow ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V0s)) \wedge \\ (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1t)))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p\ (ap \\ (c_2Epred_set_2ESING\ A_27a)\ V0s)) \Rightarrow (p\ (ap\ (c_2Epred_set_2EFINITE \\ A_27a)\ V0s)))) \end{aligned} \quad (70)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\ V0s)) \Rightarrow (\forall V1f \in (A_27b^{A_27a}). (p\ (ap\ (c_2Epred_set_2EFINITE \\ A_27b)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V1f)\ V0s)))))) \end{aligned} \quad (71)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1sos \in \\ (2^{(2^{A_27a})}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2EBIGUNION \\ A_27a)\ V1sos))) \Leftrightarrow (\exists V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V0x)\ V2s)) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a})\ V2s)\ V1sos)))))) \end{aligned} \quad (72)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}). ((\\ p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ (ap\ (c_2Epred_set_2EBIGUNION \\ A_27a)\ V0P))) \Leftrightarrow ((p\ (ap\ (c_2Epred_set_2EFINITE\ (2^{A_27a})\ V0P)) \wedge \\ (\forall V1s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (2^{A_27a}) \\ V1s)\ V0P)) \Rightarrow (p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)))))) \end{aligned} \quad (73)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \quad (74)$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (75)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (76)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (77)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \quad (79)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (80)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (81)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow ((p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (82)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))) \quad (84)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q))))) \quad (85)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p))))) \quad (86)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q))))) \quad (87)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \quad (88)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \quad \forall V0ft \in (ty_2Efmaptree_2Efmaptree \ A_27a \ A_27b).(p \ (ap \ (\\ & \quad c_2Epred_set_2EFINITE \ (ty_2Elist_2Elist \ A_27a)) \ (ap \ (c_2Epred_set_2EGSPEC \\ & \quad (ty_2Elist_2Elist \ A_27a) \ (ty_2Elist_2Elist \ A_27a)) \ (\lambda V1p \in \\ & \quad (ty_2Elist_2Elist \ A_27a).(ap \ (ap \ (c_2Epair_2E_2C \ (ty_2Elist_2Elist \\ & \quad A_27a) \ 2) \ V1p) \ (ap \ (c_2Ebool_2E_3F \ (ty_2Efmaptree_2Efmaptree \\ & \quad A_27a \ A_27b)) \ (\lambda V2ft_27 \in (ty_2Efmaptree_2Efmaptree \ A_27a \\ & \quad A_27b).(ap \ (ap \ (c_2Emin_2E_3D \ (ty_2Eoption_2Eoption \ (ty_2Efmaptree_2Efmaptree \\ & \quad A_27a \ A_27b))) \ (ap \ (ap \ (c_2Efmaptree_2Eapply_path \ A_27a \ A_27b) \\ & \quad V1p) \ V0ft)) \ (ap \ (c_2Eoption_2ESOME \ (ty_2Efmaptree_2Efmaptree \\ & \quad A_27a \ A_27b)) \ V2ft_27))))))))) \end{aligned}$$