thm_2Efmaptree_2Eitem__thm (TMTE3ZC7KoAwTC3VuNWvFW7YBD7axWvtAeo)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Efinite\_\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_\_map\_2Efmap\ A0\ A1) \tag{1}$$

Let $ty\_2Efmaptree\_2Efmaptree : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efmaptree\_2Efmaptree\ A0\ A1) \tag{2}$$

Let $c\_2Efmaptree\_2Emap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efmaptree\_2Emap\ A\_27a\ A\_27b \in ((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ (ty\_2Efmaptree\_2Efmaptree\ A\_27a\ A\_27b))^{(ty\_2Efmaptree\_2Efmaptree\ A\_27a\ A\_27b)}) \tag{3}$$

Let $c\_2Efmaptree\_2Eitem : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efmaptree\_2Eitem\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Efmaptree\_2Efmaptree\ A\_27a\ A\_27b)}) \tag{4}$$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{5}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (6)$$

Let $c\_2Efmaptree\_2EtoF : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27key.nonempty\ A\_27key \Rightarrow \forall A\_27value.nonempty\ A\_27value \Rightarrow c\_2Efmaptree\_2EtoF\ A\_27key\ A\_27value \in (((ty\_2Eoption\_2Eoption\ A\_27value)^{(ty\_2Elist\_2Elist\ A\_27key)})^{(ty\_2Efmaptree\_2Efmaptree\ A\_27key\ A\_27value)}) \quad (7)$$

Let $c\_2Efinite\_\_map\_2Eo\_\_f : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow c\_2Efinite\_\_map\_2Eo\_\_f\ A\_27a\ A\_27b\ A\_27c \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27c)^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)})^{(A\_27c^{A\_27b})}) \quad (8)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (9)$$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (10)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \quad (11)$$

Definition 10 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (12)$$

Definition 11 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Efinite\_\_map\_2Efmap\_\_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)}) \quad (13)$$

Let $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EOUTL\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \quad (14)$$

Definition 12 *We define* c_2Efinite__map_2EFAPPLY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_ma$

Let $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EISL\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \quad (15)$$

Definition 13 *We define* c_2Efinite__map_2EFDOM *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_map$

Definition 14 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 17 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Elist\_2Elist\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2Elist\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})})^{A\_27b})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (16)$$

Definition 18 *We define* c_2Efmaptree_2Econstruct *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in A\_27a.\lambda V1kfm \in ($

Let $c\_2Efmaptree\_2EfromF : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27key.nonempty\ A\_27key \Rightarrow \forall A\_27value.nonempty\ A\_27value \Rightarrow c\_2Efmaptree\_2EfromF\ A\_27key\ A\_27value \in ((ty\_2Efmaptree\_2Efmaptree\ A\_27key\ A\_27value)^{((ty\_2Eoption\_2Eoption\ A\_27value)^{(ty\_2Elist\_2Elist\ A\_27key)})}) \quad (17)$$

Definition 19 *We define* c_2Efmaptree_2EFTNode *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0i \in A\_27b.\lambda V1fm \in (ty\_$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0i1 \in A\_27b.(\forall V1f1 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ (ty\_2Efmaptree\_2Efmaptree\ A\_27a\ A\_27b)).(\forall V2i2 \in A\_27b.(\forall V3f2 \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ (ty\_2Efmaptree\_2Efmaptree\ A\_27a\ A\_27b)).(((ap\ (ap\ (\mathsf{c\_2Efmaptree\_2EFTNode}\ A\_27a\ A\_27b)\ V0i1)\ V1f1) = (ap\ (ap\ (\mathsf{c\_2Efmaptree\_2EFTNode}\ A\_27a\ A\_27b)\ V2i2)\ V3f2)) \Leftrightarrow ((V0i1 = V2i2) \wedge (V1f1 = V3f2)))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0ft \in (ty\_2Efmaptree\_2Efmaptree\ A\_27a\ A\_27b).(V0ft = (ap\ (ap\ (\mathsf{c\_2Efmaptree\_2EFTNode}\ A\_27a\ A\_27b)\ (ap\ (c\_2Efmaptree\_2Eitem\ A\_27a\ A\_27b)\ V0ft))\ (ap\ (c\_2Efmaptree\_2Emap\ A\_27a\ A\_27b)\ V0ft)))) \tag{20}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0i \in A\_27a.(\forall V1fm \in (ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ (ty\_2Efmaptree\_2Efmaptree\ A\_27b\ A\_27a)).((ap\ (c\_2Efmaptree\_2Eitem\ A\_27b\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Efmaptree\_2EFTNode}\ A\_27b\ A\_27a)\ V0i)\ V1fm)) = V0i)))$$