thm_2Efmaptree_2Erelrec__ind (TMQ3jWGDfTSRdXRto4YKQ81bt9p7QWpsGqm)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $ty\_2Efinite\_\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_\_map\_2Efmap\ A0\ A1) \tag{3}$$

Let $c\_2Efinite\_\_map\_2Efmap\_\_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)}) \tag{4}$$

Let $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EOUTL\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{5}$$

Definition 7 *We define* c_2Efinite__map_2EFAPPLY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_map$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Let $c\_2Esum\_2EISL : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Esum\_2EISL\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{6}$$

Definition 9 *We define* c_2Efinite__map_2EFDOM *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_map\_2$

Let $ty\_2Eoption\_2Eoption : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{7}$$

Let $ty\_2Elist\_2Elist : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{8}$$

Let $ty\_2Efmaptree\_2Efmaptree : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Efmaptree\_2Efmaptree\ A0\ A1) \tag{9}$$

Let $c\_2Efmaptree\_2EtoF : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27key.nonempty\ A\_27key{\Rightarrow}\forall A\_27value.nonempty\ A\_27value{\Rightarrow}c\_2Efmaptree\_2EtoF\ A\_27key\ A\_27value \in (((ty\_2Eoption\_2Eoption\ A\_27value)^{(ty\_2Elist\_2Elist\ A\_27key)})^{(ty\_2Efmaptree\_2Efmaptree\ A\_27key\ A\_27value)}) \tag{10}$$

Let $c\_2Efinite\_\_map\_2Eo\_\_f : \iota{\Rightarrow}\iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}\forall A\_27c.nonempty\ A\_27c{\Rightarrow}c\_2Efinite\_\_map\_2Eo\_\_f\ A\_27a\ A\_27b\ A\_27c \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27c)^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)})^{(A\_27c^{A\_27b})}) \tag{11}$$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A\wedge p$ *of type* $\iota{\Rightarrow}\iota$.

Definition 11 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $c\_2Esum\_2EABS\_\_sum : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{12}$$

Definition 13 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (13)$$

Definition 14 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a:\iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 17 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a:\iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Elist\_2Elist\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Elist\_2Elist\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})})^{A\_27b})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (14)$$

Definition 18 *We define* c_2Efmaptree_2Econstruct *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0a \in A\_27a.\lambda V1kfm \in ($

Let $c\_2Efmaptree\_2EfromF : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27key.nonempty\ A\_27key \Rightarrow \forall A\_27value.nonempty\ A\_27value \Rightarrow c\_2Efmaptree\_2EfromF\ A\_27key\ A\_27value \in ((ty\_2Efmaptree\_2Efmaptree\ A\_27key\ A\_27value)^{((ty\_2Eoption\_2Eoption\ A\_27value)^{(ty\_2Elist\_2Elist\ A\_27key)})}) \quad (15)$$

Definition 19 *We define* c_2Efmaptree_2EFTNode *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0i \in A\_27b.\lambda V1fm \in (ty\_$

Definition 20 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a:\iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 21 *We define* c_2Efmaptree_2Erelrec *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.(\lambda V0h \in (((A\_27c^{(ty}$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \quad (18)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x) \Rightarrow (p\ V1y)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V0x) \wedge (p\ V2z)) \Rightarrow ((p\ V1y) \wedge (p\ V3w)))))))) \tag{19}$$

Assume the following.

$$(\forall V0y \in 2.(\forall V1x \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0y) \Rightarrow (p\ V1x)) \wedge ((p\ V2z) \Rightarrow (p\ V3w))) \Rightarrow (((p\ V1x) \Rightarrow (p\ V2z)) \Rightarrow ((p\ V0y) \Rightarrow (p\ V3w)))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Rightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{22}$$

Theorem 1

$$\begin{gathered}
\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c \\
nonempty\ A_27c \Rightarrow (\forall V0h \in (((A_27c^{(ty_2Efinite__map_2Efmap\ A_27b\ (ty_2Efmaptree_2Efmaptree\ A_27b\ A_27a))} \\
(\forall V1relrec_27 \in ((2^{A_27c})^{(ty_2Efmaptree_2Efmaptree\ A_27b\ A_27a)} \\
((\forall V2i \in A_27a.(\forall V3fm \in (ty_2Efinite__map_2Efmap \\
A_27b\ (ty_2Efmaptree_2Efmaptree\ A_27b\ A_27a)).(\forall V4rfm \\
(ty_2Efinite__map_2Efmap\ A_27b\ A_27c).((((ap\ (\mathsf{c_2Efinite__map_2E} \\
A_27b\ A_27c)\ V4rfm) = (ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ (ty_2Efmaptr \\
A_27b\ A_27a))\ V3fm)) \wedge (\forall V5d \in A_27b.((p\ (ap\ (ap\ (\mathsf{c_2Ebool_2E} \\
A_27b)\ V5d)\ (ap\ (\mathsf{c_2Efinite__map_2EFDOM}\ A_27b\ (ty_2Efmaptree_2E \\
A_27b\ A_27a))\ V3fm))) \Rightarrow (p\ (ap\ (ap\ V1relrec_27\ (ap\ (ap\ (\mathsf{c_2Efinite__map} \\
A_27b\ (ty_2Efmaptree_2Efmaptree\ A_27b\ A_27a))\ V3fm)\ V5d) \\
(ap\ (\mathsf{c_2Efinite__map_2EFAPPLY}\ A_27b\ A_27c)\ V4rfm)\ V5d))))) \\
(p\ (ap\ (ap\ V1relrec_27\ (ap\ (ap\ (\mathsf{c_2Efmaptree_2EFTNode}\ A_27b\ A \\
V2i)\ V3fm))\ (ap\ (ap\ (ap\ V0h\ V2i)\ V4rfm)\ V3fm))))))) \Rightarrow (\forall V \\
(ty_2Efmaptree_2Efmaptree\ A_27b\ A_27a).(\forall V7a1 \in A_27c \\
((p\ (ap\ (ap\ (ap\ (\mathsf{c_2Efmaptree_2Erelrec}\ A_27a\ A_27b\ A_27c)\ V0h)\ V \\
V7a1)) \Rightarrow (p\ (ap\ (ap\ V1relrec_27\ V6a0)\ V7a1))))))))
\end{gathered}$$