thm_2Efmaptree_2Erelrec__rules (TMc9ooisQhGJNvqNMchDomiYqB5xaJxZvU5)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $ty\_2Efinite\_\_map\_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efinite\_\_map\_2Efmap\ A0\ A1) \tag{3}$$

Let $c\_2Efinite\_\_map\_2Efmap\_\_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efinite\_\_map\_2Efmap\_\_REP\ A\_27a\ A\_27b \in (((ty\_2Esum\_2Esum\ A\_27b\ ty\_2Eone\_2Eone)^{A\_27a})^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)}) \tag{4}$$

Let $c\_2Esum\_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EOUTL\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{5}$$

Definition 7 *We define* c_2Efinite__map_2EFAPPLY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_map$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Let $c\_2Esum\_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EISL\ A\_27a\ A\_27b \in (2^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}) \tag{6}$$

Definition 9 *We define* c_2Efinite__map_2EFDOM *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (ty\_2Efinite\_\_map\_2$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{7}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{8}$$

Let $ty\_2Efmaptree\_2Efmaptree : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efmaptree\_2Efmaptree\ A0\ A1) \tag{9}$$

Let $c\_2Efmaptree\_2EtoF : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27key.nonempty\ A\_27key \Rightarrow \forall A\_27value.nonempty\ A\_27value \Rightarrow c\_2Efmaptree\_2EtoF\ A\_27key\ A\_27value \in (((ty\_2Eoption\_2Eoption\ A\_27value)^{(ty\_2Elist\_2Elist\ A\_27key)})^{(ty\_2Efmaptree\_2Efmaptree\ A\_27key\ A\_27value)}) \tag{10}$$

Let $c\_2Efinite\_\_map\_2Eo\_\_f : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow c\_2Efinite\_\_map\_2Eo\_\_f\ A\_27a\ A\_27b\ A\_27c \in (((ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27c)^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27a\ A\_27b)})^{(A\_27c^{A\_27b})}) \tag{11}$$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Eone_2Eone *to be* $(ap\ (\text{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{12}$$

Definition 13 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a\in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{13}$$

Definition 14 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a:\iota.(\lambda V0t\in 2.(\lambda V1t1\in A\_27a.(\lambda V2t2\in A\_27a.($

Definition 16 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 17 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a:\iota.\lambda V0x\in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Elist\_2Elist\_\_CASE:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Elist\_2Elist\_\_CASE\ A\_27a\ A\_27b\in (((A\_27b^{((A\_27b^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a})})^{A\_27b})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{14}$$

Definition 18 *We define* c_2Efmaptree_2Econstruct *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0a\in A\_27a.\lambda V1kfm\in ($

Let $c\_2Efmaptree\_2EfromF:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27key.nonempty\ A\_27key\Rightarrow\forall A\_27value.nonempty\ A\_27value\Rightarrow c\_2Efmaptree\_2EfromF\ A\_27key\ A\_27value\in ((ty\_2Efmaptree\_2Efmaptree\ A\_27key\ A\_27value)^{((ty\_2Eoption\_2Eoption\ A\_27value)^{(ty\_2Elist\_2Elist\ A\_27key)})}) \tag{15}$$

Definition 19 *We define* c_2Efmaptree_2EFTNode *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0i\in A\_27b.\lambda V1fm\in (ty\_$

Definition 20 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a:\iota.(\lambda V0P\in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 21 *We define* c_2Efmaptree_2Erelrec *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.(\lambda V0h\in (((A\_27c^{(ty-}$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$(\forall V0t1\in 2.(\forall V1t2\in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{17}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{18}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1y \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0x)\Rightarrow(p\ V1y))\wedge((p\ V2z)\Rightarrow(p\ V3w)))\Rightarrow(((p\ V0x)\wedge(p\ V2z))\Rightarrow((p\ V1y)\wedge(p\ V3w)))))))) \tag{19}$$

Assume the following.

$$(\forall V0y \in 2.(\forall V1x \in 2.(\forall V2z \in 2.(\forall V3w \in 2.((((p\ V0y)\Rightarrow(p\ V1x))\wedge((p\ V2z)\Rightarrow(p\ V3w)))\Rightarrow(((p\ V1x)\Rightarrow(p\ V2z))\Rightarrow((p\ V0y)\Rightarrow(p\ V3w)))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x))\Rightarrow(p\ (ap\ V1Q\ V2x))))\Rightarrow((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x)))\Rightarrow(\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x))\Rightarrow(p\ (ap\ V1Q\ V2x))))\Rightarrow((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x)))\Rightarrow(\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{22}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c$$
$$nonempty\ A\_27c\Rightarrow(\forall V0h \in (((A\_27c^{(ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ (ty\_2Efmaptree\_2Efmaptree\ A\_27b\ A\_27a))}$$
$$(\forall V1i \in A\_27a.(\forall V2fm \in (ty\_2Efinite\_\_map\_2Efmap$$
$$A\_27b\ (ty\_2Efmaptree\_2Efmaptree\ A\_27b\ A\_27a)).(\forall V3rfm$$
$$(ty\_2Efinite\_\_map\_2Efmap\ A\_27b\ A\_27c).((((ap\ (\mathsf{c\_2Efinite\_\_map\_2E}$$
$$A\_27b\ A\_27c)\ V3rfm) = (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27b\ (ty\_2Efmaptr$$
$$A\_27b\ A\_27a))\ V2fm))\wedge(\forall V4d \in A\_27b.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2E}$$
$$A\_27b)\ V4d)\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFDOM}\ A\_27b\ (ty\_2Efmaptree\_2E$$
$$A\_27b\ A\_27a))\ V2fm)))\Rightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Efmaptree\_2Erelrec}\ A$$
$$A\_27b\ A\_27c)\ V0h)\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFAPPLY}\ A\_27b\ (ty\_2Efmaptr$$
$$A\_27b\ A\_27a))\ V2fm)\ V4d))\ (ap\ (ap\ (\mathsf{c\_2Efinite\_\_map\_2EFAPPLY}$$
$$A\_27c)\ V3rfm)\ V4d))))))\Rightarrow(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Efmaptree\_2Ereln}$$
$$A\_27a\ A\_27b\ A\_27c)\ V0h)\ (ap\ (ap\ (\mathsf{c\_2Efmaptree\_2EFTNode}\ A\_27b$$
$$V1i)\ V2fm))\ (ap\ (ap\ (ap\ V0h\ V1i)\ V3rfm)\ V2fm))))))))$$