

thm_2Efmsp_2EFMSP__FDOMSUB (TMZsHrTeZrsDyfYr6FpJxhDsJh9ERZ1497a)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ecombin_2EC$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.(\lambda V0f \in ((A.27c^{A.27b})^{A.27a}))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efinite_map_2Efmap\ A0\ A1) \tag{3}$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Efinite_map_2Efmap_REP\ A.27a\ A.27b \in (((ty_2Esum_2Esum\ A.27b\ ty_2Eone_2Eone)^{A.27a})^{(ty_2Efinite_map_2Efmap\ A.27a\ A.27b)}) \tag{4}$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Esum_2EISL\ A.27a\ A.27b \in (2^{(ty_2Esum_2Esum\ A.27a\ A.27b)}) \tag{5}$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 6 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2E_21$

Definition 7 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2E_21$

Definition 8 We define c_2Ebool_2E2F to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A-27b})^{A-27a})^2}) \end{aligned} \quad (6)$$

Definition 12 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$

Let $c_2Efinite_map_2E fmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Efinite_map_2E fmap_ABS \\ A_27a A_27b \in ((ty_2Efinite_map_2E fmap A_27a A_27b)^{(ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A-27a}}) \end{aligned} \quad (7)$$

Definition 13 We define $c_2Efinite_map_2E FEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Efinite_map_2E$

Definition 14 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2F)$.

Definition 15 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A-27a}). (ap V1f V0x)))$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod \\ A0 A1) \end{aligned} \quad (8)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A-27b})^{A-27a}}) \end{aligned} \quad (9)$$

Definition 17 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (10)$$

Definition 18 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2E$

Definition 19 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2ET)$.

Definition 20 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2E$

Definition 21 We define $c_2Epred_set_2EOMPL$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap (ap (c_2Epred_set_2E$

Let $c_2Efinite_map_2EDRESTRICT : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efinite_map_2EDRESTRICT \\ A_27a A_27b \in (((ty_2Efinite_map_2Efmap A_27a A_27b)^{(2^{A_27a})})^{(ty_2Efinite_map_2Efmap A_27a A_27b)}) \end{aligned} \quad (11)$$

Definition 22 We define $c_2Efinite_map_2Efdmsub$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0fm \in (ty_2Efinite_map_2E$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Esum_2EOUTL \\ A_27a A_27b \in (A_27a^{(ty_2Esum_2Esum A_27a A_27b)}) \end{aligned} \quad (12)$$

Definition 23 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2E$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (13)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in \\ ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \end{aligned} \quad (14)$$

Definition 24 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a))$

Definition 25 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_2Esum_2EABS$

Definition 26 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_2Eoption_2Eoption_ABS$

Definition 27 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 28 We define $c_2Efinite_map_2EFLOOKUP$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2E$

Let $ty_2Esptree_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Esptree_2Espt\ A0) \quad (15)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (16)$$

Let $c_2Esptree_2Elookup : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Elookup\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)} ty_2Enum_2Enum)) \quad (17)$$

Definition 29 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 30 We define $c_2Eoption_2EOPTREL$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). \lambda V1x$

Definition 31 We define c_2Efmsp_2EFMSP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0AN \in ((2^{ty_2Enum_2Enum})^{A_27a})$

Let $c_2Eoption_2EIS_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_SOME\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (18)$$

Let $c_2Eoption_2EIS_NONE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2EIS_NONE\ A_27a \in (2^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (19)$$

Definition 32 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1x \in A_27a. (ap\ (a$

Let $c_2Esptree_2Edelete : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Esptree_2Edelete\ A_27a \in (((ty_2Esptree_2Espt\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)} ty_2Enum_2Enum)) \quad (20)$$

Definition 33 We define $c_2Etransfer_2Eright_unique$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). (a$

Definition 34 We define $c_2Etransfer_2Eleft_unique$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). (a$

Definition 35 We define $c_2Etransfer_2Ebi_unique$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). (ap$

Definition 36 We define $c_2Etransfer_2EFUN_REL$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda A_27d : \iota. \lambda V0A$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (28)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((p V0P) \wedge (\forall V2x \in A_27a. (p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). (((\forall V2x \in A_27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A_27a. (p (ap V1Q V3x))))))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (36)$$

Assume the following.

$$2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow 2. (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c.2Ebool_2ECOND A_27a) V0P) V2x) V4y) = (ap (ap (ap (c.2Ebool_2ECOND A_27a) V1Q) V3x_27) V5y_27)))))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap (ap (ap (c.2Ebool_2ECOND A_27a) c.2Ebool_2ET) V0t1) V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap (ap (ap (c.2Ebool_2ECOND A_27a) c.2Ebool_2EF) V2t1) V3t2) = V3t2)))) \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27c^{A.27b})^{A.27a}). (\forall V1x \in \\ A.27b. (\forall V2y \in A.27a. ((ap\ (ap\ (ap\ (c.2Ecombin_2EC\ A.27a\ A.27b \\ A.27c)\ V0f)\ V1x)\ V2y) = (ap\ (ap\ V0f\ V2y)\ V1x)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0fm \in (ty_2Efinite_map_2Efm\ A.27a\ A.27b). (\forall V1k \in \\ A.27a. ((ap\ (c.2Efinite_map_2EFDOM\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Efinite_map_2Efdomsub \\ A.27a\ A.27b)\ V0fm)\ V1k)) = (ap\ (ap\ (c.2Epred_set_2EDELETE\ A.27a) \\ (ap\ (c.2Efinite_map_2EFDOM\ A.27a\ A.27b)\ V0fm))\ V1k)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0fm \in (ty_2Efinite_map_2Efm\ A.27a\ A.27b). (\forall V1k \in \\ A.27a. ((ap\ (ap\ (c.2Efinite_map_2EFAPPLY\ A.27a\ A.27b)\ (ap\ (ap \\ (c.2Efinite_map_2Efdomsub\ A.27a\ A.27b)\ V0fm)\ V1k))\ V1k) = (ap \\ (ap\ (c.2Efinite_map_2EFAPPLY\ A.27a\ A.27b)\ (c.2Efinite_map_2EFEMPTY \\ A.27a\ A.27b))\ V1k)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0fm \in (ty_2Efinite_map_2Efm\ A.27a\ A.27b). (\forall V1k1 \in \\ A.27a. (\forall V2k2 \in A.27a. ((ap\ (ap\ (c.2Efinite_map_2EFAPPLY \\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Efinite_map_2Efdomsub\ A.27a\ A.27b)\ V0fm) \\ V1k1))\ V2k2) = (ap\ (ap\ (ap\ (c.2Ebool_2ECOND\ A.27b)\ (ap\ (ap\ (c.2Emin_2E_3D \\ A.27a)\ V1k1)\ V2k2))\ (ap\ (ap\ (c.2Efinite_map_2EFAPPLY\ A.27a\ A.27b) \\ (c.2Efinite_map_2EFEMPTY\ A.27a\ A.27b))\ V2k2))\ (ap\ (ap\ (c.2Efinite_map_2EFAPPLY \\ A.27a\ A.27b)\ V0fm)\ V2k2)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\neg((c.2Eoption_2ENONE \\ A.27a) = (ap\ (c.2Eoption_2ESOME\ A.27a)\ V0x)))) \quad (44)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1X \in (\\
& \quad ty_2Eoption_2Eoption\ A_27a). (\forall V2x \in A_27a. (((ap\ (ap\ (\\
& \quad \quad ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ V1X)\ (\\
& \quad \quad c_2Eoption_2ENONE\ A_27a)) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow ((p\ V0P) \Rightarrow \\
& (p\ (ap\ (c_2Eoption_2EIS_NONE\ A_27a)\ V1X)))) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND \\
& \quad (ty_2Eoption_2Eoption\ A_27a))\ V0P)\ (c_2Eoption_2ENONE\ A_27a)) \\
& \quad V1X) = (c_2Eoption_2ENONE\ A_27a)) \Leftrightarrow ((p\ (ap\ (c_2Eoption_2EIS_SOME \\
& \quad A_27a)\ V1X)) \Rightarrow (p\ V0P))) \wedge (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption \\
& \quad A_27a))\ V0P)\ V1X)\ (c_2Eoption_2ENONE\ A_27a)) = (ap\ (c_2Eoption_2ESOME \\
& \quad A_27a)\ V2x)) \Leftrightarrow ((p\ V0P) \wedge (V1X = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V2x)))) \wedge \\
& \quad (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Eoption_2Eoption\ A_27a)) \\
& \quad V0P)\ (c_2Eoption_2ENONE\ A_27a))\ V1X) = (ap\ (c_2Eoption_2ESOME \\
& \quad A_27a)\ V2x)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1X = (ap\ (c_2Eoption_2ESOME\ A_27a) \\
& \quad V2x)))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1x \in \\
& \quad A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x) \\
& (ap\ (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap \\
& \quad (c_2Ebool_2EIN\ A_27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y))))))
\end{aligned} \tag{46}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg(\\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{56}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{61}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t \in (ty_2Esptree_2Espt \\
& A.27a). (\forall V1k1 \in ty_2Enum_2Enum. (\forall V2k2 \in ty_2Enum_2Enum. \\
& ((ap (ap (c_2Esptree_2Elookup \ A.27a) \ V1k1) (ap (ap (c_2Esptree_2Edelete \\
& A.27a) \ V2k2) \ V0t)) = (ap (ap (ap (c_2Ebool_2ECOND (ty_2Eoption_2Eoption \\
& A.27a)) (ap (ap (c_2Emin_2E_3D \ ty_2Enum_2Enum) \ V1k1) \ V2k2)) (c_2Eoption_2ENONE \\
& A.27a)) (ap (ap (c_2Esptree_2Elookup \ A.27a) \ V1k1) \ V0t))))))
\end{aligned} \tag{62}$$

Theorem 1
$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow \forall A_{27c}. \\ & \text{nonempty } A_{27c} \Rightarrow (\forall V0AN \in ((2^{ty_2Enum_2Enum})^{A_{27a}}). (\forall V1BC \in \\ & ((2^{A_{27c}})^{A_{27b}}). ((p (ap (c_2Etransfer_2Ebi_unique A_{27a} ty_2Enum_2Enum) \\ V0AN)) \Rightarrow (p (ap (ap (ap (ap (c_2Etransfer_2EFUN_REL (ty_2Efinite_map_2E fmap \\ A_{27a} A_{27b}) (ty_2Esptree_2Espt A_{27c}) ((ty_2Efinite_map_2E fmap \\ A_{27a} A_{27b})^{A_{27a}}) ((ty_2Esptree_2Espt A_{27c})^{ty_2Enum_2Enum})) \\ (ap (ap (c_2Efmsp_2EFMSP A_{27a} A_{27b} A_{27c}) V0AN) V1BC)) (ap (ap \\ (c_2Etransfer_2EFUN_REL A_{27a} ty_2Enum_2Enum (ty_2Efinite_map_2E fmap \\ A_{27a} A_{27b}) (ty_2Esptree_2Espt A_{27c}) V0AN) (ap (ap (c_2Efmsp_2EFMSP \\ A_{27a} A_{27b} A_{27c}) V0AN) V1BC))) (c_2Efinite_map_2Efdoms sub A_{27a} \\ A_{27b}) (ap (c_2Ecombin_2EC ty_2Enum_2Enum (ty_2Esptree_2Espt \\ A_{27c}) (ty_2Esptree_2Espt A_{27c})) (c_2Esptree_2Edelete A_{27c}))))))))) \end{aligned}$$