

thm_2Efmsp_2EFMSP__bitotal (TMPNGMgo- HEW2LuW1KgLsuGknJhkf8g21ZnF)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}}))$

Definition 5 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t)).$

Definition 6 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow_p Q)$ of type ι .

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2.V2t))$

Definition 8 We define `c_2Ebool_2ECOND` to be $\lambda A_{27a} : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_{27a}. (\lambda V2t2 \in A_{27a}. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V3t3 \in 2.V3t3))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty \ ty_2Eone_2Eone \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty \ A0 \Rightarrow \forall A1. nonempty \ A1 \Rightarrow nonempty \ (ty_2Esum_2Esum \ A0 \ A1) \tag{2}$$

Let $ty_2Efinite_map_2Efmap : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty \ A0 \Rightarrow \forall A1. nonempty \ A1 \Rightarrow nonempty \ (ty_2Efinite_map_2Efmap \ A0 \ A1) \tag{3}$$

Let $c_2Efinite_map_2Efmap_REP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2Efmap_REP \\ & A_27a\ A_27b \in (((ty_2Esum_2Esum\ A_27b\ ty_2Eone_2Eone)^{A_27a})^{(ty_2Efinite_map_2Efmap\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Let $c_2Esum_2EOUTL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EOUTL \\ & A_27a\ A_27b \in (A_27a^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (5)$$

Definition 9 We define $c_2Efinite_map_2EFAPPLY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFUN_FMAP$

Let $c_2Efinite_map_2EFUN_FMAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUN_FMAP \\ & A_27a\ A_27b \in (((ty_2Efinite_map_2Efmap\ A_27a\ A_27b)^{(2^{A_27a})})^{(A_27b^{A_27a})}) \end{aligned} \quad (6)$$

Let $c_2Esum_2EISL : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EISL \\ & A_27a\ A_27b \in (2^{(ty_2Esum_2Esum\ A_27a\ A_27b)}) \end{aligned} \quad (7)$$

Definition 10 We define $c_2Efinite_map_2EFDOM$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFDOM$

Definition 11 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone.$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ & A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (8)$$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (9)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ & ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (10)$$

Definition 14 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c_2Eoption_2ENONE$

Definition 15 We define c_Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap (c_Esum_2EABS$

Definition 16 We define $c_Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap (c_Eoption_2Eoption_2EABS$

Definition 17 We define c_Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Definition 18 We define $c_Efinite_map_2EFLOOKUP$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (ty_2Efinite_map_2EFLOOKUP$

Let $ty_2Esptree_2Espt : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Esptree_2Espt A0) \quad (11)$$

Let $c_2Esptree_2ELN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Esptree_2ELN A_27a \in (ty_2Esptree_2Espt A_27a) \quad (12)$$

Let $c_2Efinite_map_2Efmap_ABS : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Efinite_map_2Efmap_ABS A_27a A_27b \in ((ty_2Efinite_map_2Efmap A_27a A_27b)^{(ty_2Esum_2Esum A_27b ty_2Eone_2Eone)^{A_27a}}) \quad (13)$$

Definition 19 We define $c_2Efinite_map_2EFEMPTY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap (c_2Efinite_map_2EFEMPTY$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (14)$$

Let $c_2Esptree_2Einsert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Esptree_2Einsert A_27a \in (((ty_2Esptree_2Espt A_27a)^{(ty_2Esptree_2Espt A_27a)})^{A_27a})^{ty_2Enum_2Enum} \quad (15)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (16)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (17)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (18)$$

Definition 20 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})^{A_27b})^{A_27a}$.
 Let $c_2Efinite_map_2EFUPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Efinite_map_2EFUPDATE\ A_27a\ A_27b \in \\ & ((ty_2Efinite_map_2Efmmap\ A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})^{(ty_2Efinite_map_2EFUPDATE\ A_27a\ A_27b)} \end{aligned} \quad (19)$$

Let $c_2Esptree_2Elookup : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow c_2Esptree_2Elookup\ A_27a \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esptree_2Espt\ A_27a)})^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \end{aligned} \quad (20)$$

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a\ V0P))))$.

Definition 22 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_3F\ V2t)\ V1t2))))$.

Definition 23 We define $c_2Eoption_2EOPTREL$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). \lambda V1x \in (A_27b)^{A_27a}. (ap\ V0R\ (ap\ (c_2Eoption_2EOPTREL\ A_27a\ A_27b)\ V1x))$.

Definition 24 We define c_2Efmsp_2EFMSP to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0AN \in ((2^{ty_2Enum_2Enum\ A_27a})^{A_27b})^{A_27c}$.

Let $c_2Eoption_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Eoption_2ETHE\ A_27a \in (A_27a^{(ty_2Eoption_2Eoption\ A_27a)}) \quad (21)$$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in \\ & ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})})^{A_27a} \end{aligned} \quad (22)$$

Definition 25 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2ETET\ A_27a\ V0x)$.

Definition 26 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27b)^{A_27a}. (ap\ V0f\ (ap\ (c_2Ebool_2ETET\ A_27a\ V1s)))$.

Definition 27 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27b)^{A_27a}. (ap\ V0f\ (ap\ (c_2Ebool_2ETET\ A_27a\ V1s)))$.

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in \\ & ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})})^{A_27a} \end{aligned} \quad (23)$$

Definition 28 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$.

Definition 29 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27b)^{A_27a}. (ap\ V0f\ (ap\ (c_2Ebool_2ETET\ A_27a\ V1s)))$.

Definition 30 We define $c_2Epred_set_2ELINV_OPT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27b)^{A_27a}. (ap\ V0f\ (ap\ (c_2Ebool_2ETET\ A_27a\ V1s)))$.

Definition 31 We define $c_2Epred_set_2ELINV$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (A_27b)^{A_27a}. (ap\ V0f\ (ap\ (c_2Ebool_2ETET\ A_27a\ V1s)))$.

Definition 32 We define $c_2\text{Epred_set_2EBIJ}$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a}).$

Definition 33 We define $c_2\text{Epred_set_2EINSERT}$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap (c_2Ebool_2EF)$

Definition 34 We define $c_2\text{Epred_set_2EEMPTY}$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF).$

Definition 35 We define $c_2\text{Epred_set_2EFINITE}$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). (ap (c_2Ebool_2E_21 (c_2Ebool_2EF)))$

Let $c_2\text{Esptree_2Edomain} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2\text{Esptree_2Edomain } A_27a \in ((2^{ty_2Enum_2Enum})^{(ty_2Esptree_2Esptree_2Edomain A_27a)}) \quad (24)$$

Definition 36 We define $c_2\text{Etransfer_2Erigh_unique}$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). (ap (c_2Ebool_2EF))$

Definition 37 We define $c_2\text{Etransfer_2Eleft_unique}$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). (ap (c_2Ebool_2EF))$

Definition 38 We define $c_2\text{Etransfer_2Ebi_unique}$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). (ap (c_2Ebool_2EF))$

Definition 39 We define $c_2\text{Etransfer_2Esurj}$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). (ap (c_2Ebool_2EF))$

Definition 40 We define $c_2\text{Etransfer_2Etotal}$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). (ap (c_2Ebool_2EF))$

Definition 41 We define $c_2\text{Etransfer_2Ebitotal}$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). (ap (c_2Ebool_2EF))$

Definition 42 We define $c_2\text{Etransfer_2EFUN_REL}$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda A_27d : \iota. \lambda V0A \in ((2^{A_27b})^{A_27a}). (ap (c_2Ebool_2EF))$

Let $c_2\text{Etransfer_2EPAIR_REL} : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow \forall A_27c. \\ & nonempty A_27c \Rightarrow \forall A_27d. nonempty A_27d \Rightarrow c_2\text{Etransfer_2EPAIR_REL} \\ & A_27a A_27b A_27c A_27d \in (((2^{(ty_2Epair_2Eprod A_27b A_27d)})^{(ty_2Epair_2Eprod A_27a A_27c)})^{(2^{A_27d})^{A_27c}}) \end{aligned} \quad (25)$$

Assume the following.

$$True \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg (p V0t)))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (34)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF) \\ V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in \\ A_27a. (p\ (ap\ V0P\ V1x)))))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow \\ ((\forall V3x \in A_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A_27a. (p\ (\\ ap\ V1Q\ V4x))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ 2. (((\forall V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in \\ A_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). (((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow (\forall V3x \in \\ A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ 2. ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ A_27a. (p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (47)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee (p \ V1B) \wedge (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \quad (48)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (49)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Leftrightarrow (p \ V1t2)) \Leftrightarrow (((p \ V0t1) \Rightarrow (p \ V1t2)) \wedge ((p \ V1t2) \Rightarrow (p \ V0t1))))) \quad (50)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge ((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \\ & V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \\ & V5y_27)))))) \end{aligned} \quad (52)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0a \in A_27a.(\exists V1x \in A_27a.(V1x = V0a))) \quad (53)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in \\ & A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (\\ & ap \ V0P \ V1a)))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \forall V0P \in (2^{(ty_2Efinite_map_2E fmap \ A_27a \ A_27b)}).(((p \ (\\ & (ap \ V0P \ (c_2Efinite_map_2EFEMPTY \ A_27a \ A_27b))) \wedge (\forall V1f \in \\ & (ty_2Efinite_map_2E fmap \ A_27a \ A_27b).((p \ (ap \ V0P \ V1f)) \Rightarrow (\forall V2x \in \\ & A_27a.(\forall V3y \in A_27b.((\neg(p \ (ap \ (ap \ (c_2Ebool_2EIN \ A_27a) \\ & V2x) \ (ap \ (c_2Efinite_map_2EFDOM \ A_27a \ A_27b) \ V1f)))) \Rightarrow (p \ (ap \ V0P \\ & (ap \ (ap \ (c_2Efinite_map_2EFUPDATE \ A_27a \ A_27b) \ V1f) \ (ap \ (ap \ (c_2Epair_2E_2C \\ & A_27a \ A_27b) \ V2x) \ V3y)))))) \Rightarrow (\forall V4f \in (ty_2Efinite_map_2E fmap \\ & A_27a \ A_27b).(p \ (ap \ V0P \ V4f)))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (A.27b^{A.27a}).(\forall V1P \in (2^{A.27a}).((p\ (ap\ (c.2Epred_set_2EFINITE \\
& A.27a)\ V1P)) \Rightarrow ((ap\ (c.2Efinite_map_2EFDOM\ A.27a\ A.27b)\ (ap\ (\\
& ap\ (c.2Efinite_map_2EFUN_FMAP\ A.27a\ A.27b)\ V0f)\ V1P)) = V1P) \wedge \\
& (\forall V2x \in A.27a.((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V2x)\ V1P)) \Rightarrow \\
& ((ap\ (ap\ (c.2Efinite_map_2EFAPPLY\ A.27a\ A.27b)\ (ap\ (ap\ (c.2Efinite_map_2EFUN_FMAP \\
& A.27a\ A.27b)\ V0f)\ V1P))\ V2x) = (ap\ V0f\ V2x)))))) \\
& (56)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \forall V0f \in (A.27b^{A.27a}).(\forall V1s \in (2^{A.27a}).((p\ (ap\ (c.2Epred_set_2EFINITE \\
& A.27a)\ V1s)) \Rightarrow ((ap\ (c.2Efinite_map_2EFDOM\ A.27a\ A.27b)\ (ap\ (ap \\
& (c.2Efinite_map_2EFUN_FMAP\ A.27a\ A.27b)\ V0f)\ V1s)) = V1s)))) \\
& (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0AN \in ((2^{ty.2Enum.2Enum})^{A.27a}).(\forall V1BC \in \\
& ((2^{A.27c})^{A.27b}).(p\ (ap\ (ap\ (ap\ (ap\ (c.2Efmsp.2EFMSP\ A.27a\ A.27b \\
& A.27c)\ V0AN)\ V1BC))\ (c.2Efinite_map_2EFEMPTY\ A.27a\ A.27b))\ (c.2Esptree.2ELN \\
& A.27c)))) \\
& (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0AN \in ((2^{ty.2Enum.2Enum})^{A.27a}).(\forall V1BC \in \\
& ((2^{A.27c})^{A.27b}).((p\ (ap\ (c.2Etransfer.2Ebi_unique\ A.27a\ ty.2Enum.2Enum) \\
& V0AN)) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ (c.2Etransfer.2EFUN_REL\ (ty.2Efinite_map.2Efmap \\
& A.27a\ A.27b)\ (ty.2Esptree.2Espt\ A.27c)\ ((ty.2Efinite_map.2Efmap \\
& A.27a\ A.27b)^{(ty.2Epair.2Eprod\ A.27a\ A.27b)})\ ((ty.2Esptree.2Espt \\
& A.27c)^{(ty.2Epair.2Eprod\ ty.2Enum.2Enum\ A.27c)}))\ (ap\ (ap\ (c.2Efmsp.2EFMSP \\
& A.27a\ A.27b\ A.27c)\ V0AN)\ V1BC))\ (ap\ (ap\ (c.2Etransfer.2EFUN_REL \\
& (ty.2Epair.2Eprod\ A.27a\ A.27b)\ (ty.2Epair.2Eprod\ ty.2Enum.2Enum \\
& A.27c)\ (ty.2Efinite_map.2Efmap\ A.27a\ A.27b)\ (ty.2Esptree.2Espt \\
& A.27c))\ (ap\ (ap\ (c.2Etransfer.2EPAIR_REL\ A.27a\ ty.2Enum.2Enum \\
& A.27b\ A.27c)\ V0AN)\ V1BC))\ (ap\ (ap\ (c.2Efmsp.2EFMSP\ A.27a\ A.27b\ A.27c) \\
& V0AN)\ V1BC)))\ (c.2Efinite_map.2EFUPDATE\ A.27a\ A.27b))\ (\lambda V2sp \in \\
& (ty.2Esptree.2Espt\ A.27c).(ap\ (c.2Epair.2EUNCURRY\ ty.2Enum.2Enum \\
& A.27c\ (ty.2Esptree.2Espt\ A.27c))\ (\lambda V3n \in ty.2Enum.2Enum.(\\
& \lambda V4v \in A.27c.(ap\ (ap\ (ap\ (c.2Esptree.2Einsert\ A.27c)\ V3n)\ V4v) \\
& V2sp))))))))) \\
& (59)
\end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\ A_27a).((V0opt = (c_2Eoption_2ENONE\ A_27a)) \vee (\exists V1x \in A_27a. \\ (V0opt = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V1x)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ A_27a.(((ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x) = (ap\ (c_2Eoption_2ESOME \\ A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\neg((c_2Eoption_2ENONE \\ A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((ap\ (c_2Eoption_2ETHE \\ A_27a)\ (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)) = V0x))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in \\ A_27b.(((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}).(\forall V1t \in \\ (2^{A_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}).(\forall V1v \in \\ A_27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b.((ap\ (ap\ (c_2Epair_2E_2C \\ A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x))))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A_27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A_27a)))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\
& \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in \\
& \quad (2^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EINJ\ A_27a\ A_27b)\ V0f) \\
& \quad V1s)\ V2t)) \Rightarrow (\forall V3x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a) \\
& \quad V3x)\ V1s)) \Rightarrow ((ap\ (ap\ (ap\ (c_2Epred_set_2ELINV\ A_27a\ A_27b)\ V0f) \\
& \quad V1s)\ (ap\ V0f\ V3x)) = V3x)))))))))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1s \in (2^{A_27a}). (\forall V2t \in \\
& \quad (2^{A_27b}). ((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EBIJ\ A_27a\ A_27b)\ V0f) \\
& \quad V1s)\ V2t)) \Rightarrow (\forall V3x \in A_27b. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b) \\
& \quad V3x)\ V2t)) \Rightarrow ((ap\ V0f\ (ap\ (ap\ (ap\ (c_2Epred_set_2ELINV\ A_27a\ A_27b) \\
& \quad V0f)\ V1s)\ V3x)) = V3x)))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a) \\
& \quad V0s)) \Rightarrow (\forall V1f \in (A_27b^{A_27a}). (p\ (ap\ (c_2Epred_set_2EFINITE \\
& \quad A_27b)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V1f)\ V0s))))))
\end{aligned} \tag{71}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{75}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow ((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (76)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (\\ &(p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(\\ &p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\ &((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (\\ &(p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\ &(\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))) \end{aligned} \quad (78)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (\\ &(p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\ &((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))) \end{aligned} \quad (79)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (\\ &(p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\ &\neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p)))))))) \end{aligned} \quad (80)$$

Assume the following.

$$\begin{aligned} &(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\ &(p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \end{aligned} \quad (81)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \quad (82)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V0p)))) \quad (84)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \quad (85)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \quad (86)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0t \in (ty.2Esptree.2Espt \\ & A.27a).(p \ (ap \ (c.2Epred_set.2EFINITE \ ty.2Enum.2Enum) \ (ap \ (c.2Esptree.2Edomain \\ & A.27a) \ V0t)))) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0k \in ty.2Enum.2Enum.(\\ & \forall V1t \in (ty.2Esptree.2Espt \ A.27a).(((ap \ (ap \ (c.2Esptree.2Ellookup \\ & A.27a) \ V0k) \ V1t) = (c.2Eoption.2ENONE \ A.27a)) \Leftrightarrow (\neg(p \ (ap \ (ap \ (c.2Ebool.2EIN \\ & ty.2Enum.2Enum) \ V0k) \ (ap \ (c.2Esptree.2Edomain \ A.27a) \ V1t)))))) \end{aligned} \quad (88)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c. \\ & nonempty \ A.27c \Rightarrow \forall A.27d.nonempty \ A.27d \Rightarrow (\forall V0AB \in (\\ & (2^{A.27b})^{A.27a}).(\forall V1CD \in ((2^{A.27d})^{A.27c}).(\forall V2f \in \\ & (A.27c^{A.27a}).(\forall V3g \in (A.27d^{A.27b}).(\forall V4a \in A.27a. \\ & (\forall V5b \in A.27b.(((p \ (ap \ (ap \ (ap \ (c.2Etransfer.2EFUN_REL \\ & A.27a \ A.27b \ A.27c \ A.27d) \ V0AB) \ V1CD) \ V2f) \ V3g)) \wedge (p \ (ap \ (ap \ V0AB \ V4a) \\ & V5b)))) \Rightarrow (p \ (ap \ (ap \ V1CD \ (ap \ V2f \ V4a)) \ (ap \ V3g \ V5b)))))))) \end{aligned} \quad (89)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c. \\ & nonempty \ A.27c \Rightarrow \forall A.27d.nonempty \ A.27d \Rightarrow (\forall V0AB \in (\\ & (2^{A.27b})^{A.27a}).(\forall V1CD \in ((2^{A.27d})^{A.27c}).(\forall V2a \in \\ & A.27a.(\forall V3c \in A.27c.(\forall V4b \in A.27b.(\forall V5d \in A.27d. \\ & ((p \ (ap \ (ap \ (ap \ (ap \ (c.2Etransfer.2EPAIR_REL \ A.27a \ A.27b \ A.27c \\ & A.27d) \ V0AB) \ V1CD) \ (ap \ (ap \ (c.2Epair.2E_2C \ A.27a \ A.27c) \ V2a) \ V3c)) \\ & (ap \ (ap \ (c.2Epair.2E_2C \ A.27b \ A.27d) \ V4b) \ V5d)))) \Leftrightarrow ((p \ (ap \ (ap \ V0AB \\ & V2a) \ V4b)) \wedge (p \ (ap \ (ap \ V1CD \ V3c) \ V5d)))))) \end{aligned} \quad (90)$$

Theorem 1

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c. \\ & nonempty \ A.27c \Rightarrow (\forall V0AN \in ((2^{ty.2Enum.2Enum})^{A.27a}).(\forall V1BC \in \\ & ((2^{A.27c})^{A.27b}).(((p \ (ap \ (c.2Etransfer.2Ebitotal \ A.27a \ ty.2Enum.2Enum) \\ & V0AN)) \wedge ((p \ (ap \ (c.2Etransfer.2Ebitotal \ A.27b \ A.27c) \ V1BC)) \wedge (\\ & p \ (ap \ (c.2Etransfer.2Ebi_unique \ A.27a \ ty.2Enum.2Enum) \ V0AN)))) \Rightarrow \\ & (p \ (ap \ (c.2Etransfer.2Ebitotal \ (ty.2Efinite_map.2Efmap \ A.27a \\ & A.27b) \ (ty.2Esptree.2Espt \ A.27c)) \ (ap \ (ap \ (c.2Efmsp.2EFMSP \ A.27a \\ & A.27b \ A.27c) \ V0AN) \ V1BC)))) \end{aligned}$$