thm_2Efrac_2EFRAC__ADD__ASSOC (TMFgKoJYpKnnyC32Khasga8JYwtXZigd66Z)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{3}$$

Let $c\_2Efrac\_2Erep\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{4}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{5}$$

Definition 7 *We define* c_2Efrac_2Efrac__dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ ty$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{6}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{7}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty_2Einteger_2Eint.(ap\ ($c_2Emin_2E_40 $(ty$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{8}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{9}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \tag{10}$$

Definition 10 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 11 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{11}$$

Definition 12 *We define* c_2Efrac_2Efrac__nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{12}$$

Definition 13 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{13}$$

Definition 15 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \tag{14}$$

Definition 16 *We define* c_2Efrac_2Efrac__add *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Ef$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{15}$$

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{16}$$

Definition 17 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{17}$$

Definition 18 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{18}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{19}$$

Definition 19 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 20 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{21}$$

Definition 21 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{22}$$

Definition 22 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 23 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 24 *We define* c_2Einteger_2Etint__1 *to be* $(ap\ (ap\ ($c_2Epair_2E_2C $ty\_2Enum\_2Enum\ ty\_2Enum$

Definition 25 *We define* c_2Einteger_2Eint__1 *to be* $(ap$ c_2Einteger_2Eint__ABS c_2Einteger_2Etint__1$)$.

Definition 26 *We define* c_2Einteger_2Etint__0 *to be* $(ap\ (ap\ ($c_2Epair_2E_2C $ty\_2Enum\_2Enum\ ty\_2Enum$

Definition 27 *We define* c_2Einteger_2Eint__0 *to be* $(ap$ c_2Einteger_2Eint__ABS c_2Einteger_2Etint__0$)$.

Let $ty\_2Ering\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ering\_2Ering\ A0) \tag{23}$$

Let $c\_2Ering\_2Erecordtype\_2Ering : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Erecordtype\_2Ering\ A\_27a \in ((((((ty\_2Ering\_2Ering\ A\_27a)^{(A\_27a^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{24}$$

Let $ty\_2Ecanonical\_2Ecanonical\_\_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A0) \tag{25}$$

Let $ty\_2EringNorm\_2Epolynom : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2EringNorm\_2Epolynom\ A0) \tag{26}$$

Let $c\_2EringNorm\_2Epolynom\_\_normalize : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EringNorm\_2Epolynom\_\_normalize\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2EringNorm\_2Epolynom\ A\_27a)})^{(ty\_2Ering\_2Ering\ A} \tag{27}$$

Definition 28 *We define* c_2EintegerRing_2Eint__polynom__normalize *to be* $(ap\ (c\_2EringNorm\_2Epolynom$

Let $c\_2Ering\_2Ering\_\_RM : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RM\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{28}$$

Let $c\_2Ering\_2Ering\_\_RP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_RP\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{29}$$

Let $c\_2Ering\_2Ering\_\_R1 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R1\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{30}$$

Let $c\_2Ering\_2Ering\_\_R0 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ering\_2Ering\_\_R0\ A\_27a \in (A\_27a^{(ty\_2Ering\_2Ering\ A\_27a)}) \tag{31}$$

Let $ty\_2Esemi\_\_ring\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A0) \tag{32}$$

Let $c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Esemi\_\_ring\_2Erecordtype\_2Esemi\_\_ring\ A\_27a \in (((((ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)^{((A\_27a^{A\_27a})^{A\_27a})})^{((A\_27a^{A\_27a})^{A\_27a})})^{A\_27a})^{A\_27a}) \tag{33}$$

Definition 29 *We define* c_2Ering_2Esemi__ring__of *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(a$

Let $ty\_2Equote\_2Evarmap : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Equote\_2Evarmap\ A0) \tag{34}$$

Let $c\_2Ecanonical\_2Eics\_\_aux : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Eics\_\_aux\ A\_27a \in ((((A\_27a^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{A\_27a})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_r} \tag{35}$$

Definition 30 *We define* c_2EringNorm_2Er__ics__aux *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).($

Definition 31 *We define* c_2EintegerRing_2Eint__r__ics__aux *to be* $(ap$ (c_2EringNorm_2Er__ics__aux ty_2Eint

Let $ty\_2Equote\_2Eindex : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Equote\_2Eindex \tag{36}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{37}$$

Let $c\_2Ecanonical\_2Einterp\_\_m : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Einterp\_\_m\ A\_27a \in ((((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{A\_27a})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring} \tag{38}$$

Definition 32 *We define* c_2EringNorm_2Er__interp__m *to be* $\lambda A\_27a:\iota.\lambda V0r\in(ty\_2Ering\_2Ering\ A\_27a)$

Definition 33 *We define* c_2EintegerRing_2Eint__r__interp__m *to be* $(ap$ (c_2EringNorm_2Er__interp__m ty_2E

Let $c\_2Ecanonical\_2Einterp\_\_vl:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ecanonical\_2Einterp\_\_vl\ A\_27a\in (((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27a)}) \tag{39}$$

Definition 34 *We define* c_2EringNorm_2Er__interp__vl *to be* $\lambda A\_27a:\iota.\lambda V0r\in(ty\_2Ering\_2Ering\ A\_27a)$

Definition 35 *We define* c_2EintegerRing_2Eint__r__interp__vl *to be* $(ap$ (c_2EringNorm_2Er__interp__vl ty_2E

Let $c\_2Ecanonical\_2Eivl\_\_aux:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ecanonical\_2Eivl\_\_aux\ A\_27a\in ((((A\_27a^{(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex)})^{ty\_2Equote\_2Eindex})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2} \tag{40}$$

Definition 36 *We define* c_2EringNorm_2Er__ivl__aux *to be* $\lambda A\_27a:\iota.\lambda V0r\in(ty\_2Ering\_2Ering\ A\_27a).($

Definition 37 *We define* c_2EintegerRing_2Eint__r__ivl__aux *to be* $(ap$ (c_2EringNorm_2Er__ivl__aux ty_2Einte

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ecanonical\_2Ecanonical\_\_sum\_\_simplify\ A\_27a\in(((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Esemi\_\_ri} \tag{41}$$

Definition 38 *We define* c_2EringNorm_2Er__canonical__sum__simplify *to be* $\lambda A\_27a:\iota.\lambda V0r\in(ty\_2Ering\_2$

Definition 39 *We define* c_2EintegerRing_2Eint__r__canonical__sum__simplify *to be* $(ap$ (c_2EringNorm_2Er__canonical__sum__simplify $ty\_2Einteger\_2Eint)$ $(ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2E$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_prod:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ecanonical\_2Ecanonical\_\ A\_27a\in((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Ecanonic} \tag{42}$$

Definition 40 *We define* c_2EringNorm_2Er__canonical__sum__prod *to be* $\lambda A\_27a:\iota.\lambda V0r\in(ty\_2Ering\_2E$

Definition 41 *We define* c_2EintegerRing_2Eint__r__canonical__sum__prod *to be* $(ap$ (c_2EringNorm_2Er__canonical__sum__prod $ty\_2Einteger\_2Eint)$ $(ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Ereco$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar3 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_$$
$$A\_27a \in (((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2E} \quad (43)$$

Definition 42 *We define* c_2EringNorm_2Er__canonical__sum__scalar3 *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2$

Definition 43 *We define* c_2EintegerRing_2Eint__r__canonical__sum__scalar3 *to be* $(ap$ (c_2EringNorm_2Er__canonical__sum__scalar3 $ty\_2Einteger\_2Eint)$ $(ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erec$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar2 : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_sum$$
$$A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2El} \quad (44)$$

Definition 44 *We define* c_2EringNorm_2Er__canonical__sum__scalar2 *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2$

Definition 45 *We define* c_2EintegerRing_2Eint__r__canonical__sum__scalar2 *to be* $(ap$ (c_2EringNorm_2Er__canonical__sum__scalar2 $ty\_2Einteger\_2Eint)$ $(ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erec$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_sum\_\_scalar$$
$$A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{A\_27a})^{(ty\_2Es} \quad (45)$$

Definition 46 *We define* c_2EringNorm_2Er__canonical__sum__scalar *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2E$

Definition 47 *We define* c_2EintegerRing_2Eint__r__canonical__sum__scalar *to be* $(ap$ (c_2EringNorm_2Er__canonical__sum__scalar $ty\_2Einteger\_2Eint)$ $(ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erec$

Let $c\_2Ecanonical\_2Evarlist\_\_insert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Evarlist\_\_i$$
$$A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2El} \quad (46)$$

Definition 48 *We define* c_2EringNorm_2Er__varlist__insert *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_2$

Definition 49 *We define* c_2EintegerRing_2Eint__r__varlist__insert *to be* $(ap$ (c_2EringNorm_2Er__varlist__inser

Let $c\_2Ecanonical\_2Emonom\_\_insert : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Emonom$$
$$A\_27a \in (((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2E} \quad (47)$$

Definition 50 *We define* c_2EringNorm_2Er__monom__insert *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A$

Definition 51 *We define* c_2EintegerRing_2Eint__r__monom__insert *to be* $(ap$ (c_2EringNorm_2Er__monom__in

Let $c\_2Ecanonical\_2ENil\_\_monom : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ENil\_\_monom\ A\_27a \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a) \tag{48}$$

Let $c\_2Ecanonical\_2ECons\_\_varlist : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_varlist\ A\_27a \in (((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Eli} \tag{49}$$

Let $c\_2Ecanonical\_2ECons\_\_monom : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2ECons\_\_monom\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Elist\_2Elist\ ty\_2Eq} \tag{50}$$

Let $c\_2Ecanonical\_2Ecanonical\_\_sum\_\_merge : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ecanonical\_2Ecanonical\_\_s\ A\_27a \in ((((ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Ecanonic} \tag{51}$$

Definition 52 *We define* c_2EringNorm_2Er__canonical__sum__merge *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2I$

Definition 53 *We define* c_2EintegerRing_2Eint__r__canonical__sum__merge *to be* $(ap$ (c_2EringNorm_2Er__canonical__sum__merge $ty\_2Einteger\_2Eint)$ $(ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erec$

Let $c\_2Equote\_2EEmpty\_\_vm : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Equote\_2EEmpty\_\_vm\ A\_27a \in (ty\_2Equote\_2Evarmap\ A\_27a) \tag{52}$$

Let $c\_2Equote\_2ENode\_\_vm : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Equote\_2ENode\_\_vm\ A\_27a \in ((((ty\_2Equote\_2Evarmap\ A\_27a)^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{A\_27a}) \tag{53}$$

Let $c\_2EringNorm\_2EPopp : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EringNorm\_2EPopp\ A\_27a \in ((ty\_2EringNorm\_2Epolynom\ A\_27a)^{(ty\_2EringNorm\_2Epolynom\ A\_27a)}) \tag{54}$$

Let $c\_2EringNorm\_2EPmult : \iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EringNorm\_2EPmult\ A\_27a \in (((ty\_2EringNorm\_2Epolynom\ A\_27a)^{(ty\_2EringNorm\_2Epolynom\ A\_27a)})^{(ty\_2EringNorm\_2Epolynom\ A\_27a)}) \tag{55}$$

Let $c\_2EringNorm\_2EPplus : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2EringNorm\_2EPplus\ A\_27a \in (((ty\_2EringNorm\_2Epolynom\ A\_27a)^{(ty\_2EringNorm\_2Epolynom\ A\_27a)})^{(ty\_2EringNorm\_2Epolynom\ A\_27a)}) \quad (56)$$

Let $c\_2Equote\_2Evarmap\_\_find : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Equote\_2Evarmap\_\_find\ A\_27a \in ((A\_27a^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{ty\_2Equote\_2Eindex}) \quad (57)$$

Let $c\_2EringNorm\_2EPvar : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2EringNorm\_2EPvar\ A\_27a \in ((ty\_2EringNorm\_2Epolynom\ A\_27a)^{ty\_2Equote\_2Eindex}) \quad (58)$$

Let $c\_2EringNorm\_2EPconst : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2EringNorm\_2EPconst\ A\_27a \in ((ty\_2EringNorm\_2Epolynom\ A\_27a)^{A\_27a}) \quad (59)$$

Definition 54 *We define* c_2EringNorm_2Epolynom__simplify *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A$

Definition 55 *We define* c_2EintegerRing_2Eint__polynom__simplify *to be* $(ap$ (c_2EringNorm_2Epolynom__sin

Let $c\_2Ecanonical\_2Einterp\_\_cs : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ecanonical\_2Einterp\_\_cs\ A\_27a \in (((A\_27a^{(ty\_2Ecanonical\_2Ecanonical\_\_sum\ A\_27a)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Esemi\_\_ring\_2Esemi\_\_ring\ A\_27} \quad (60)$$

Definition 56 *We define* c_2EringNorm_2Er__interp__cs *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a)$

Definition 57 *We define* c_2EintegerRing_2Eint__r__interp__cs *to be* $(ap$ (c_2EringNorm_2Er__interp__cs ty_2E

Let $c\_2EringNorm\_2Einterp\_\_p : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2EringNorm\_2Einterp\_\_p\ A\_27a \in (((A\_27a^{(ty\_2EringNorm\_2Epolynom\ A\_27a)})^{(ty\_2Equote\_2Evarmap\ A\_27a)})^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (61)$$

Definition 58 *We define* c_2EintegerRing_2Eint__interp__p *to be* $(ap\ (c\_2EringNorm\_2Einterp\_\_p\ ty\_2Eint$

Let $c\_2Ering\_2Ering\_\_RN : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ering\_2Ering\_\_RN\ A\_27a \in ((A\_27a^{A\_27a})^{(ty\_2Ering\_2Ering\ A\_27a)}) \quad (62)$$

Definition 59 *We define* c_2Ering_2Eis__ring *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ering\_2Ering\ A\_27a).(ap\ (ap$ c

Let $c\_2Equote\_2ERight\_\_idx : \iota$ be given. Assume the following.

$$c\_2Equote\_2ERight\_\_idx \in (ty\_2Equote\_2Eindex^{ty\_2Equote\_2Eindex}) \tag{63}$$

Let $c\_2Equote\_2ELeft\_\_idx : \iota$ be given. Assume the following.

$$c\_2Equote\_2ELeft\_\_idx \in (ty\_2Equote\_2Eindex^{ty\_2Equote\_2Eindex}) \tag{64}$$

Let $c\_2Equote\_2EEnd\_\_idx : \iota$ be given. Assume the following.

$$c\_2Equote\_2EEnd\_\_idx \in ty\_2Equote\_2Eindex \tag{65}$$

Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{66}$$

Let $c\_2Equote\_2Eindex\_\_compare : \iota$ be given. Assume the following.

$$c\_2Equote\_2Eindex\_\_compare \in ((ty\_2EternaryComparisons\_2Eordering^{ty\_2Equote\_2Eindex})^{ty\_2Equote\_2E} \tag{67}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{68}$$

Definition 60 *We define* c_2Equote_2Eindex__lt *to be* $\lambda V0i1 \in ty\_2Equote\_2Eindex.\lambda V1i2 \in ty\_2Equote\_2$

Let $c\_2EternaryComparisons\_2Enum2ordering : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2Enum2ordering \in (ty\_2EternaryComparisons\_2Eordering^{ty\_2Enum\_2Enum} \tag{69}$$

Definition 61 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2EternaryComparisons\_2Eordering2num : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2Eordering2num \in (ty\_2Enum\_2Enum^{ty\_2EternaryComparisons\_2Eordering}) \tag{70}$$

Definition 62 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 63 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 64 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 65 *We define* c_2EternaryComparisons_2Eordering__CASE *to be* $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Eternary$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \quad (71)$$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \quad (72)$$

Let $c\_2EternaryComparisons\_2Elist\_\_compare : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2EternaryComparisons A\_27a\ A\_27b \in (((ty\_2EternaryComparisons\_2Eordering^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((ty \quad (73)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (74)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (75)$$

Let $c\_2EternaryComparisons\_2Elist\_\_merge : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EternaryComparisons\_2Elist\_\_merge\ A\_27a \in ((((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)})^{((2^{A\_27a})^{A\_27a})}) \quad (76)$$

Assume the following.

$$True \quad (77)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (78)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \quad (79)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \quad (80)$$

Assume the following.

$$(\forall V0f \in ty\_2Efrac\_2Efrac.(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ V0f)))) \tag{81}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b)) \Rightarrow ((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V0a)))) \tag{82}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b)) \Rightarrow ((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V1b)))) \tag{83}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0a)) \Rightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0a)\ V1b))))))) \tag{84}$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow ((p\ (ap\ (\mathsf{c\_2Ering\_2Eis\_\_ring}\ ty\_2Einteger\_2Eint) \\
(ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ering\_2Erecordtype\_2Ering\ ty\_2Einteger\_2Eint) \\
\mathsf{c\_2Einteger\_2Eint\_\_0})\ \mathsf{c\_2Einteger\_2Eint\_\_1})\ \mathsf{c\_2Einteger\_2Eint\_\_add}) \\
\mathsf{c\_2Einteger\_2Eint\_\_mul})\ \mathsf{c\_2Einteger\_2Eint\_\_neg})))\wedge((\forall V0vm \in \\
(ty\_2Equote\_2Evarmap\ ty\_2Einteger\_2Eint).(\forall V1p \in (ty\_2EringNorm\_2Epolynom \\
ty\_2Einteger\_2Eint).((ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p} \\
V0vm)\ V1p) = (ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_r\_\_interp\_\_cs}\ V0vm) \\
(ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_polynom\_\_simplify}\ V1p)))))\wedge(((\\
(\forall V2vm \in (ty\_2Equote\_2Evarmap\ ty\_2Einteger\_2Eint).(\forall V3c \in \\
ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p} \\
V2vm)\ (ap\ (c\_2EringNorm\_2EPconst\ ty\_2Einteger\_2Eint)\ V3c)) = \\
V3c)))\wedge((\forall V4vm \in (ty\_2Equote\_2Evarmap\ ty\_2Einteger\_2Eint). \\
(\forall V5i \in ty\_2Equote\_2Eindex.((ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p} \\
V4vm)\ (ap\ (c\_2EringNorm\_2EPvar\ ty\_2Einteger\_2Eint)\ V5i)) = (ap \\
(ap\ (c\_2Equote\_2Evarmap\_\_find\ ty\_2Einteger\_2Eint)\ V5i)\ V4vm))))\wedge \\
((\forall V6vm \in (ty\_2Equote\_2Evarmap\ ty\_2Einteger\_2Eint).(\\
\forall V7p1 \in (ty\_2EringNorm\_2Epolynom\ ty\_2Einteger\_2Eint). \\
(\forall V8p2 \in (ty\_2EringNorm\_2Epolynom\ ty\_2Einteger\_2Eint). \\
((ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p}\ V6vm)\ (ap\ (ap\ (c\_2EringNorm\_2EPplus \\
ty\_2Einteger\_2Eint)\ V7p1)\ V8p2)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add} \\
(ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p}\ V6vm)\ V7p1))\ (ap\ (ap \\
\mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p}\ V6vm)\ V8p2))))))\wedge((\forall V9vm \in \\
(ty\_2Equote\_2Evarmap\ ty\_2Einteger\_2Eint).(\forall V10p1 \in (\\
ty\_2EringNorm\_2Epolynom\ ty\_2Einteger\_2Eint).(\forall V11p2 \in \\
(ty\_2EringNorm\_2Epolynom\ ty\_2Einteger\_2Eint).((ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p} \\
V9vm)\ (ap\ (ap\ (c\_2EringNorm\_2EPmult\ ty\_2Einteger\_2Eint)\ V10p1) \\
V11p2)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p} \\
V9vm)\ V10p1))\ (ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p}\ V9vm) \\
V11p2))))))\wedge(\forall V12vm \in (ty\_2Equote\_2Evarmap\ ty\_2Einteger\_2Eint). \\
(\forall V13p1 \in (ty\_2EringNorm\_2Epolynom\ ty\_2Einteger\_2Eint). \\
((ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p}\ V12vm)\ (ap\ (c\_2EringNorm\_2EPopp \\
ty\_2Einteger\_2Eint)\ V13p1)) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap \\
(ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_interp\_\_p}\ V12vm)\ V13p1)))))))))\wedge \\
((\forall V14x \in A\_27a.(\forall V15v2 \in (ty\_2Equote\_2Evarmap\ A\_27a). \\
(\forall V16v1 \in (ty\_2Equote\_2Evarmap\ A\_27a).((ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find \\
A\_27a)\ c\_2Equote\_2EEnd\_\_idx)\ (ap\ (ap\ (ap\ (c\_2Equote\_2ENode\_\_vm \\
A\_27a)\ V14x)\ V16v1)\ V15v2)) = V14x))))\wedge((\forall V17x \in A\_27a.(\\
\forall V18v2 \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V19v1 \in (\\
ty\_2Equote\_2Evarmap\ A\_27a).(\forall V20i1 \in ty\_2Equote\_2Eindex. \\
((ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find\ A\_27a)\ (ap\ c\_2Equote\_2ERight\_\_idx \\
V20i1))\ (ap\ (ap\ (ap\ (c\_2Equote\_2ENode\_\_vm\ A\_27a)\ V17x)\ V19v1)\ V18v2)) = \\
(ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find\ A\_27a)\ V20i1)\ V18v2))))))\wedge \\
((\forall V21x \in A\_27a.(\forall V22v2 \in (ty\_2Equote\_2Evarmap\ A\_27a). \\
(\forall V23v1 \in (ty\_2Equote\_2Evarmap\ A\_27a).(\forall V24i1 \in \\
ty\_2Equote\_2Eindex.((ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find\ A\_27a) \\
(ap\ c\_2Equote\_2ELeft\_\_idx\ V24i1))\ (ap\ (ap\ (ap\ (c\_2Equote\_2ENode\_\_vm \\
A\_27a)\ V21x)\ V23v1)\ V22v2)) = (ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find \\
A\_27a)\ V24i1)\ V23v1))))))\wedge(\forall V25i \in ty\_2Equote\_2Eindex. \\
((ap\ (ap\ (c\_2Equote\_2Evarmap\_\_find\ A\_27a)\ V25i)\ (c\_2Equote\_2EEmpty\_\_vm \\
A\_27a)) = (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A\_27a)\ (\lambda V26x \in A\_27a.\mathsf{c\_2Ebool\_2ET}))))))))\wedge \\
(((\forall V27t2 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ ty\_2Einteger\_2Eint). \\
(\forall V28t1 \in (ty\_2Ecanonical\_2Ecanonical\_\_sum\ ty\_2Einteger\_2Eint). \\
(\forall V29l2 \in (ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V30l1 \in \\
(ty\_2Elist\_2Elist\ ty\_2Equote\_2Eindex).(\forall V31c2 \in ty\_2Einteger\_2Eint. \\
(\forall V32c1 \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2EintegerRing\_2Eint\_\_r\_\_canonical\_\_sum\_\_merge} \\
(ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom\ ty\_2Einteger\_2Eint) \\
V32c1)\ V30l1)\ V28t1))\ (ap\ (ap\ (ap\ (c\_2Ecanonical\_2ECons\_\_monom \\
ty\_2Einteger\_2Eint)\ V31c2)\ V29l2)\ V27t2)) = (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CA} \\
(ty\_2Ecanonical\_2Ecanonical\_\_sum\ ty\_2Einteger\_2Eint))\ (ap
\end{gathered}
$$

Assume the following.

$$
\begin{gathered}
(((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare\ c\_2Equote\_2EEnd\_\_idx) \\
c\_2Equote\_2EEnd\_\_idx) = c\_2EternaryComparisons\_2EEQUAL) \land (\\
(\forall V0v10 \in ty\_2Equote\_2Eindex.((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare \\
c\_2Equote\_2EEnd\_\_idx)\ (ap\ c\_2Equote\_2ELeft\_\_idx\ V0v10)) = c\_2EternaryComparisons\_2ELESS) \\
((\forall V1v11 \in ty\_2Equote\_2Eindex.((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare \\
c\_2Equote\_2EEnd\_\_idx)\ (ap\ c\_2Equote\_2ERight\_\_idx\ V1v11)) = c\_2EternaryComparisons\_2ELESS \\
((\forall V2v2 \in ty\_2Equote\_2Eindex.((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare \\
(ap\ c\_2Equote\_2ELeft\_\_idx\ V2v2))\ c\_2Equote\_2EEnd\_\_idx) = c\_2EternaryComparisons\_2EGREATE \\
((\forall V3v3 \in ty\_2Equote\_2Eindex.((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare \\
(ap\ c\_2Equote\_2ERight\_\_idx\ V3v3))\ c\_2Equote\_2EEnd\_\_idx) = c\_2EternaryComparisons\_2EGREATE \\
((\forall V4n\_27 \in ty\_2Equote\_2Eindex.(\forall V5m\_27 \in ty\_2Equote\_2Eindex. \\
((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare\ (ap\ c\_2Equote\_2ELeft\_\_idx \\
V4n\_27))\ (ap\ c\_2Equote\_2ELeft\_\_idx\ V5m\_27)) = (ap\ (ap\ c\_2Equote\_2Eindex\_\_compare \\
V4n\_27)\ V5m\_27)))) \land ((\forall V6n\_27 \in ty\_2Equote\_2Eindex.(\forall V7m\_27 \in \\
ty\_2Equote\_2Eindex.((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare\ (ap \\
c\_2Equote\_2ELeft\_\_idx\ V6n\_27))\ (ap\ c\_2Equote\_2ERight\_\_idx\ V7m\_27)) = \\
c\_2EternaryComparisons\_2ELESS))) \land ((\forall V8n\_27 \in ty\_2Equote\_2Eindex. \\
(\forall V9m\_27 \in ty\_2Equote\_2Eindex.((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare \\
(ap\ c\_2Equote\_2ERight\_\_idx\ V8n\_27))\ (ap\ c\_2Equote\_2ERight\_\_idx \\
V9m\_27)) = (ap\ (ap\ c\_2Equote\_2Eindex\_\_compare\ V8n\_27)\ V9m\_27)))) \land \\
(\forall V10n\_27 \in ty\_2Equote\_2Eindex.(\forall V11m\_27 \in ty\_2Equote\_2Eindex. \\
((ap\ (ap\ c\_2Equote\_2Eindex\_\_compare\ (ap\ c\_2Equote\_2ERight\_\_idx \\
V10n\_27))\ (ap\ c\_2Equote\_2ELeft\_\_idx\ V11m\_27)) = c\_2EternaryComparisons\_2EGREATER)))))))
\end{gathered}
\tag{86}
$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0v0 \in A\_27a.(\forall V1v1 \in \\
A\_27a.(\forall V2v2 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE} \\
A\_27a)\ c\_2EternaryComparisons\_2ELESS)\ V0v0)\ V1v1)\ V2v2) = V0v0)))) \land \\
((\forall V3v0 \in A\_27a.(\forall V4v1 \in A\_27a.(\forall V5v2 \in A\_27a. \\
((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}\ A\_27a) \\
c\_2EternaryComparisons\_2EEQUAL)\ V3v0)\ V4v1)\ V5v2) = V4v1)))) \land \\
(\forall V6v0 \in A\_27a.(\forall V7v1 \in A\_27a.(\forall V8v2 \in A\_27a. \\
((ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE}\ A\_27a) \\
c\_2EternaryComparisons\_2EGREATER)\ V6v0)\ V7v1)\ V8v2) = V8v2))))))
\end{gathered}
\tag{87}
$$

Assume the following.

$$
\begin{gathered}
((\forall V0x \in \mathit{ty\_2EternaryComparisons\_2Eordering}.((V0x = V0x) \Leftrightarrow \\
\mathit{True})) \wedge (((\mathit{c\_2EternaryComparisons\_2ELESS} = \mathit{c\_2EternaryComparisons\_2EEQUAL}) \Leftrightarrow \\
\mathit{False}) \wedge (((\mathit{c\_2EternaryComparisons\_2ELESS} = \mathit{c\_2EternaryComparisons\_2EGREATER}) \Leftrightarrow \\
\mathit{False}) \wedge (((\mathit{c\_2EternaryComparisons\_2EEQUAL} = \mathit{c\_2EternaryComparisons\_2EGREATER}) \Leftrightarrow \\
\mathit{False}) \wedge (((\mathit{c\_2EternaryComparisons\_2EEQUAL} = \mathit{c\_2EternaryComparisons\_2ELESS}) \Leftrightarrow \\
\mathit{False}) \wedge (((\mathit{c\_2EternaryComparisons\_2EGREATER} = \mathit{c\_2EternaryComparisons\_2ELESS}) \Leftrightarrow \\
\mathit{False}) \wedge (((\mathit{c\_2EternaryComparisons\_2EGREATER} = \mathit{c\_2EternaryComparisons\_2EEQUAL}) \Leftrightarrow \\
\mathit{False}) \wedge ((((\mathit{ap}\ \mathit{c\_2EternaryComparisons\_2Eordering2num}\ \mathit{c\_2EternaryComparisons\_2ELESS}) = \\
\mathsf{c\_2Enum\_2E0}) \wedge (((\mathit{ap}\ \mathit{c\_2EternaryComparisons\_2Eordering2num} \\
\mathit{c\_2EternaryComparisons\_2EEQUAL}) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) \wedge ((\mathit{ap}\ \mathit{c\_2EternaryComparisons\_2Eordering2num} \\
\mathit{c\_2EternaryComparisons\_2EGREATER}) = (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \wedge (((\mathit{ap} \\
\mathit{c\_2EternaryComparisons\_2Enum2ordering}\ \mathsf{c\_2Enum\_2E0}) = \mathit{c\_2EternaryComparisons\_2ELESS}) \wedge \\
(((\mathit{ap}\ \mathit{c\_2EternaryComparisons\_2Enum2ordering}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = \mathit{c\_2EternaryComparisons\_2EEQUAL}) \wedge \\
((\mathit{ap}\ \mathit{c\_2EternaryComparisons\_2Enum2ordering}\ (\mathit{ap}\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(\mathit{ap}\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = \mathit{c\_2EternaryComparisons\_2EGREATER})))))))
\end{gathered}
\tag{88}
$$

Assume the following.

$$
\begin{gathered}
\forall \mathit{A\_27a}.\mathit{nonempty}\ \mathit{A\_27a} \Rightarrow \forall \mathit{A\_27b}.\mathit{nonempty}\ \mathit{A\_27b} \Rightarrow (\\
(\forall V0cmp \in ((\mathit{ty\_2EternaryComparisons\_2Eordering}^{\mathit{A\_27b}})^{\mathit{A\_27a}}). \\
((\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2EternaryComparisons\_2Elist\_\_compare}\ \mathit{A\_27a}\ \mathit{A\_27b}) \\
V0cmp)\ (\mathit{c\_2Elist\_2ENIL}\ \mathit{A\_27a}))\ (\mathit{c\_2Elist\_2ENIL}\ \mathit{A\_27b})) = \mathit{c\_2EternaryComparisons\_2EEQUAL}) \\
((\forall V1v9 \in (\mathit{ty\_2Elist\_2Elist}\ \mathit{A\_27b}).(\forall V2v8 \in \mathit{A\_27b}. \\
(\forall V3cmp \in ((\mathit{ty\_2EternaryComparisons\_2Eordering}^{\mathit{A\_27b}})^{\mathit{A\_27a}}). \\
((\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2EternaryComparisons\_2Elist\_\_compare}\ \mathit{A\_27a}\ \mathit{A\_27b}) \\
V3cmp)\ (\mathit{c\_2Elist\_2ENIL}\ \mathit{A\_27a}))\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Elist\_2ECONS}\ \mathit{A\_27b}) \\
V2v8)\ V1v9)) = \mathit{c\_2EternaryComparisons\_2ELESS})))) \wedge ((\forall V4v5 \in \\
(\mathit{ty\_2Elist\_2Elist}\ \mathit{A\_27a}).(\forall V5v4 \in \mathit{A\_27a}.(\forall V6cmp \in \\
((\mathit{ty\_2EternaryComparisons\_2Eordering}^{\mathit{A\_27b}})^{\mathit{A\_27a}}).((\mathit{ap}\ (\mathit{ap} \\
(\mathit{ap}\ (\mathit{c\_2EternaryComparisons\_2Elist\_\_compare}\ \mathit{A\_27a}\ \mathit{A\_27b})\ V6cmp) \\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Elist\_2ECONS}\ \mathit{A\_27a})\ V5v4)\ V4v5))\ (\mathit{c\_2Elist\_2ENIL}\ \mathit{A\_27b})) = \\
\mathit{c\_2EternaryComparisons\_2EGREATER})))) \wedge (\forall V7y \in \mathit{A\_27b}. \\
(\forall V8x \in \mathit{A\_27a}.(\forall V9l2 \in (\mathit{ty\_2Elist\_2Elist}\ \mathit{A\_27b}). \\
(\forall V10l1 \in (\mathit{ty\_2Elist\_2Elist}\ \mathit{A\_27a}).(\forall V11cmp \in ((\\
\mathit{ty\_2EternaryComparisons\_2Eordering}^{\mathit{A\_27b}})^{\mathit{A\_27a}}).((\mathit{ap}\ (\mathit{ap}\ (\\
\mathit{ap}\ (\mathit{c\_2EternaryComparisons\_2Elist\_\_compare}\ \mathit{A\_27a}\ \mathit{A\_27b})\ V11cmp) \\
(\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Elist\_2ECONS}\ \mathit{A\_27a})\ V8x)\ V10l1))\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2Elist\_2ECONS} \\
\mathit{A\_27b})\ V7y)\ V9l2)) = (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathsf{c\_2EternaryComparisons\_2Eordering\_\_CASE} \\
\mathit{ty\_2EternaryComparisons\_2Eordering})\ (\mathit{ap}\ (\mathit{ap}\ V11cmp\ V8x)\ V7y)) \\
\mathit{c\_2EternaryComparisons\_2ELESS})\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{ap}\ (\mathit{c\_2EternaryComparisons\_2Elist\_\_compare} \\
\mathit{A\_27a}\ \mathit{A\_27b})\ V11cmp)\ V10l1)\ V9l2))\ \mathit{c\_2EternaryComparisons\_2EGREATER}))))))))
\end{gathered}
\tag{89}
$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow ((\forall V0l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V1a\_\_lt \in ((2^{A\_27a})^{A\_27a}).((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ A\_27a)\ V1a\_\_lt)\ V0l1)\ (c\_2Elist\_2ENIL\ A\_27a)) = V0l1))) \wedge ((\forall V2v5 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3v4 \in A\_27a.(\forall V4a\_\_lt \in ((2^{A\_27a})^{A\_27a}).((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ A\_27a)\ V4a\_\_lt)\ (c\_2Elist\_2ENIL\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3v4)\ V2v5)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3v4)\ V2v5))))) \wedge (\forall V5y \in A\_27a.(\forall V6x \in A\_27a.(\forall V7l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V8l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V9a\_\_lt \in ((2^{A\_27a})^{A\_27a}).((ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ A\_27a)\ V9a\_\_lt)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V6x)\ V8l1))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5y)\ V7l2)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ (ap\ V9a\_\_lt\ V6x)\ V5y))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V6x)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ A\_27a)\ V9a\_\_lt)\ V8l1)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5y)\ V7l2))))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V5y)\ (ap\ (ap\ (ap\ (c\_2EternaryComparisons\_2Elist\_\_merge\ A\_27a)\ V9a\_\_lt)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V6x)\ V8l1))\ V7l2)))))))))) \tag{90}$$

Theorem 1

$$(\forall V0a \in ty\_2Efrac\_2Efrac.(\forall V1b \in ty\_2Efrac\_2Efrac.(\forall V2c \in ty\_2Efrac\_2Efrac.((ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_add}\ V0a)\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_add}\ V1b)\ V2c)) = (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_add}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_add}\ V0a)\ V1b))\ V2c)))))$$