thm_2Efrac_2EFRAC__ADD__SAVE (TMTRB6ya6hzx7tuHATFqM38eFoK1s4xRLsT)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\text{c\_2Emin\_2E\_40}\ A$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{3}$$

Let $c\_2Efrac\_2Erep\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{4}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{5}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Efrac_2Efrac__dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ ty$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{6}$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2} \tag{7}$$

Definition 7 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Einteger\_2Etint\_\_mul : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{8}$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{9}$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}} \tag{10}$$

Definition 8 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2$

Definition 9 *We define* c_2Einteger_2Eint__mul *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{11}$$

Definition 10 *We define* c_2Efrac_2Efrac__nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty$

Let $c\_2Einteger\_2Etint\_\_add : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty} \tag{12}$$

Definition 11 *We define* c_2Einteger_2Eint__add *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Intege$

Definition 12 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 13 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (13)$$

Definition 14 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \quad (14)$$

Definition 15 *We define* c_2Efrac_2Efrac__add *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2E$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (15)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (16)$$

Definition 16 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 17 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (17)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (18)$$

Definition 18 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (19)$$

Definition 19 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 20 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (20)$$

Definition 21 *We define* c_2Efrac_2Efrac__save *to be* $\lambda V0nmr \in ty\_2Einteger\_2Eint.\lambda V1dnm \in ty\_2Enur$

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum} \tag{21}$$

Definition 22 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Definition 23 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 24 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 25 *We define* c_2Einteger_2Eint__le *to be* $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2E$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{22}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{23}$$

Definition 26 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 27 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 28 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 29 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 30 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \tag{24}$$

Definition 31 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap\ \mathsf{c\_2Einteger\_2Eint}$

Definition 32 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 33 *We define* c_2Emarker_2EAC *to be* $\lambda V0b1 \in 2.\lambda V1b2 \in 2.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ V0b1)$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{25}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p))))) \tag{26}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m)) \tag{27}$$

Assume the following.

$$True \tag{28}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{29}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{34}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{37}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{39}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg (p\ V0A)) \vee (p\ V1B))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((A\_27a^{A\_27a})^{A\_27a}).(((\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((ap\ (ap\ V0f\ V1x)\ (ap\ (ap\ V0f\ V2y)\ V3z)) = (ap\ (ap\ V0f\ (ap\ (ap\ V0f\ V1x)\ V2y))\ V3z))))) \Rightarrow ((\forall V4x \in A\_27a.(\forall V5y \in A\_27a.((ap\ (ap\ V0f\ V4x)\ V5y) = (ap\ (ap\ V0f\ V5y)\ V4x)))) \Rightarrow (\forall V6x \in A\_27a.(\forall V7y \in A\_27a.(\forall V8z \in A\_27a.((ap\ (ap\ V0f\ V6x)\ (ap\ (ap\ V0f\ V7y)\ V8z)) = (ap\ (ap\ V0f\ V7y)\ (ap\ (ap\ V0f\ V6x)\ V8z))))))))) \tag{41}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b)) \Rightarrow ((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V0a)))) \tag{42}$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b)) \Rightarrow ((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V1b)))) \tag{43}$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ V0x)\ V1y)) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))))))) \quad (44)$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Einteger\_2Eint}).((\forall V1n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)))) \Leftrightarrow (\forall V2x \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V2x)) \Rightarrow (p\ (ap\ V0P\ V2x)))))) \quad (45)$$

Assume the following.

$$(\forall V0c \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.(\forall V2y \in ty\_2Einteger\_2Eint.((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0c)\ V1x))) \Rightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ V2y)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1x))) \Rightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_le}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0c))\ V2y))) \Leftrightarrow False)))))) \quad (46)$$

Assume the following.

$$(\forall V0y \in ty\_2Einteger\_2Eint.(\forall V1x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1x)\ V0y) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0y)\ V1x)))) \quad (47)$$

Assume the following.

$$(\forall V0z \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V2x)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V0z)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V2x)\ V1y))\ V0z))))) \quad (48)$$

Assume the following.

$$(\forall V0z \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V0z)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ V1y))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V2x)\ V0z)))))) \quad (49)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = V0x)) \quad (50)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V0x) = V0x)) \quad (51)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) = V0x)) \quad (52)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ V1y))\ V2z) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V1y)\ V2z)))))) \quad (53)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(\forall V2z \in ty\_2Einteger\_2Eint.(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ V2z) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V1y)\ V2z)) \Leftrightarrow (V0x = V1y))))) \quad (54)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y))))) \quad (55)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0x))\ V1y)))) \quad (56)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ V0x)\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V1y))))) \quad (57)$$

Assume the following.

$$((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \quad (58)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n))\Leftrightarrow(V0m = V1n)))) \tag{59}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))) \tag{60}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V0m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1n)) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))))) \tag{61}$$

Assume the following.

$$(\forall V0p \in ty\_2Einteger\_2Eint.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2m \in ty\_2Enum\_2Enum.(((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0p) = V0p) \land (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ V0p)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = V0p) \land (((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \land (((ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ V0p)) = V0p) \land (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m))) = (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2m))))) \land (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m)))) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Einteger\_2Eint)\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V2m)\ V1n))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1n)\ V2m))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2m)\ V1n)))))) \land (((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m))) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Einteger\_2Eint)\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2m))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V2m)\ V1n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1n)\ V2m)))))) \land ((ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2m)))) = (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2m))))))))))))))) \tag{62}$$

Assume the following.

$$
(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(
((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
\mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\Leftrightarrow
True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
\mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow
False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
\mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))))\Leftrightarrow False)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\
V0n)))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))\Leftrightarrow False)\wedge
(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\
\mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
\mathsf{c\_2Enum\_2E0})))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\
c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n)))\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\
V1m))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))\wedge(((p\ (ap\ (ap\
\mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V0n)))))\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\
V1m))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V0n)))))\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m))))\Leftrightarrow True)\wedge(((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\
V0n)))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V1m)))))\Leftrightarrow False)\wedge((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\
(ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (
ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V0n))))\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_neg}\
(ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\
V1m)))))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V0n))))))))))))))))
\tag{63}
$$

Assume the following.

$$
(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{64}
$$

Assume the following.

$$
(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{65}
$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{66}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{67}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{68}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{69}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{72}$$

Theorem 1

$$(\forall V0a1 \in ty\_2Einteger\_2Eint.(\forall V1b1 \in ty\_2Enum\_2Enum. (\forall V2a2 \in ty\_2Einteger\_2Eint.(\forall V3b2 \in ty\_2Enum\_2Enum. ((ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_add}\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_save}\ V0a1) V1b1))\ (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_save}\ V2a2)\ V3b2)) = (ap\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_save} (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add} (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_add}\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul} V0a1)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V3b2)))\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_mul} V2a2)\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ V1b1))))\ V0a1))\ V2a2))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1b1)\ V3b2))\ V1b1))\ V3b2)))))))$$