

# thm\_2Efrac\_2EFrac\_\_AINV\_\_CALCULATE (TMJxmuouaeQHkF3m6o2hZqcWaLfZiBzaShX)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{3}$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \tag{4}$$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.^{27a} : \iota.( \lambda V0P \in (2^{A.^{27a}}).(ap (ap (c\_2Emin\_2E\_3D (2^{A.^{27a}}))$

**Definition 5** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E\_40 (ty$

Let  $c\_2Einteger\_2Eint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Einteger\_2Eint}) \tag{5}$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (6)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})} \quad (7)$$

**Definition 6** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 7** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap\ c\_2Einteger\_2Eint\_eq)$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \quad (8)$$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \quad (9)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (10)$$

**Definition 8** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ ty\_2Efrac\_2Efrac))$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (11)$$

**Definition 9** We define  $c\_2Efrac\_2Efrac\_nmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty\_2Efrac\_2Efrac))$

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2)))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (12)$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b))$

Let  $c\_2Efrac\_2Eabs\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Eabs\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \quad (13)$$

**Definition 13** We define  $c\_2Efrac\_2Efrac\_ainv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_frac)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (14)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (15)$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)) \quad (17)$$

**Definition 15** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Assume the following.

$$True \quad (18)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Einteger\_2Eint. (\forall V1b \in ty\_2Einteger\_2Eint. \\ & ((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num \\ & c\_2Enum\_2E0))\ V1b)) \Rightarrow ((ap\ c\_2Efrac\_2Efrac\_nmr\ (ap\ c\_2Efrac\_2Eabs\_frac \\ & (ap\ (ap\ (c\_2Epair\_2E.2C\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint) \\ & V0a)\ V1b))) = V0a)))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Einteger\_2Eint. (\forall V1b \in ty\_2Einteger\_2Eint. \\ & ((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num \\ & c\_2Enum\_2E0))\ V1b)) \Rightarrow ((ap\ c\_2Efrac\_2Efrac\_dnm\ (ap\ c\_2Efrac\_2Eabs\_frac \\ & (ap\ (ap\ (c\_2Epair\_2E.2C\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint) \\ & V0a)\ V1b))) = V1b)))) \end{aligned} \quad (21)$$

**Theorem 1**

$$\begin{aligned} & (\forall V0a1 \in ty\_2Einteger\_2Eint. (\forall V1b1 \in ty\_2Einteger\_2Eint. \\ & ((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num \\ c\_2Enum\_2E0)) V1b1)) \Rightarrow ((ap c\_2Efrac\_2Efrac\_ainv (ap c\_2Efrac\_2Eabs\_frac \\ (ap (ap (c\_2Epair\_2E\_2C ty\_2Einteger\_2Eint ty\_2Einteger\_2Eint) \\ V0a1) V1b1))) = (ap c\_2Efrac\_2Eabs\_frac (ap (ap (c\_2Epair\_2E\_2C \\ ty\_2Einteger\_2Eint ty\_2Einteger\_2Eint) (ap c\_2Einteger\_2Eint\_neg \\ V0a1)) V1b1))))))) \end{aligned}$$