thm_2Efrac_2EFRAC__AINV__ONEONE (TMVnuphN-wxnW8XDaNVr9RmjH6izLqmjNEdB)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EONE__ONE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(ap\ (\text{c\_2}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \text{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \text{c\_2Ebool\_2EF}$

Let $ty\_2Einteger\_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$
$$\tag{2}$$

Let $ty\_2Efrac\_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{3}$$

Let $c\_2Efrac\_2Erep\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Erep\_\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint$$
$$ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{4}$$

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Let $c\_2Epair\_2ESND : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2ESND$$
$$A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$$

$$(5)$$

Definition 8 *We define* c_2Efrac_2Efrac__dnm *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ ty$

Let $c\_2Epair\_2EFST : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EFST$$
$$A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$$

$$(6)$$

Definition 9 *We define* c_2Efrac_2Efrac__nmr *to be* $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2EFST\ ty\_$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (7)$$

Let $c\_2Einteger\_2Eint\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2}$$

$$(8)$$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A\wedge$
of type $\iota\Rightarrow\iota.$

Definition 11 *We define* c_2Einteger_2Eint__REP *to be* $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ ($c_2Emin_2E_40 $(t$

Let $c\_2Einteger\_2Etint\_\_neg : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum$$
$$ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \qquad (9)$$

Let $c\_2Einteger\_2Etint\_\_eq : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum}$$

$$(10)$$

Let $c\_2Einteger\_2Eint\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_ABS\_\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}$$

$$(11)$$

Definition 12 *We define* c_2Einteger_2Eint__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_$

Definition 13 *We define* c_2Einteger_2Eint__neg *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap$ c_2Einteger_2Eint_

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{12}$$

Definition 15 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Efrac\_2Eabs\_\_frac : \iota$ be given. Assume the following.

$$c\_2Efrac\_2Eabs\_\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \tag{13}$$

Definition 16 *We define* c_2Efrac_2Efrac__ainv *to be* $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_\_fr$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{14}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{15}$$

Definition 17 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Let $c\_2Einteger\_2Eint\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Eint\_\_of\_\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \tag{16}$$

Let $c\_2Einteger\_2Etint\_\_lt : \iota$ be given. Assume the following.

$$c\_2Einteger\_2Etint\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_.}} \tag{17}$$

Definition 18 *We define* c_2Einteger_2Eint__lt *to be* $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_.$

Assume the following.
$$True \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow$$
$$True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\tag{19}$$
$$(p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow$$
$$True)) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in$$
$$A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{21}$$

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Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))))) \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (23)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))) \quad (24)$$

Assume the following.

$$(\forall V0f \in ty\_2Efrac\_2Efrac.((ap\ \mathsf{c\_2Efrac\_2Eabs\_\_frac}\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ V0f))\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ V0f))) = V0f)) \quad (25)$$

Assume the following.

$$(\forall V0a1 \in ty\_2Einteger\_2Eint.(\forall V1b1 \in ty\_2Einteger\_2Eint. (\forall V2a2 \in ty\_2Einteger\_2Eint.(\forall V3b2 \in ty\_2Einteger\_2Eint. ((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b1)) \Rightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V3b2)) \Rightarrow (((ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a1)\ V1b1)) = (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V2a2)\ V3b2))) \Leftrightarrow ((V0a1 = V2a2) \wedge (V1b1 = V3b2)))))))))) \quad (26)$$

Assume the following.

$$(\forall V0f \in ty\_2Efrac\_2Efrac.(p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Efrac\_2Efrac\_\_dnm}\ V0f)))) \quad (27)$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint. ((p\ (ap\ (ap\ \mathsf{c\_2Einteger\_2Eint\_\_lt}\ (ap\ c\_2Einteger\_2Eint\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1b)) \Rightarrow ((ap\ \mathsf{c\_2Efrac\_2Efrac\_\_nmr}\ (ap\ c\_2Efrac\_2Eabs\_\_frac\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)\ V0a)\ V1b))) = V0a)))) \quad (28)$$

Assume the following.

$$(\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint.$$
$$((p \ (ap \ (ap \ \mathsf{c\_2Einteger\_2Eint\_lt} \ (ap \ c\_2Einteger\_2Eint\_of\_num$$
$$\mathsf{c\_2Enum\_2E0})) \ V1b)) \Rightarrow ((ap \ \mathsf{c\_2Efrac\_2Efrac\_dnm} \ (ap \ c\_2Efrac\_2Eabs\_frac$$
$$(ap \ (ap \ (\mathsf{c\_2Epair\_2E\_2C} \ ty\_2Einteger\_2Eint \ ty\_2Einteger\_2Eint)$$
$$V0a) \ V1b))) = V1b))))$$

$$(29)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.$$
$$(((ap \ \mathsf{c\_2Einteger\_2Eint\_neg} \ V0x) = (ap \ \mathsf{c\_2Einteger\_2Eint\_neg} \quad (30)$$
$$V1y)) \Leftrightarrow (V0x = V1y))))$$

Theorem 1

$$(p \ (ap \ (\mathsf{c\_2Ebool\_2EONE\_ONE} \ ty\_2Efrac\_2Efrac \ ty\_2Efrac\_2Efrac)$$
$$\mathsf{c\_2Efrac\_2Efrac\_ainv}))$$