

thm\_2Efrac\_2EFRAC\_\_AINV\_\_SAVE  
(TMZtzaw5tnFgUrHEYNUfqU4LdXwEmc95EzX)

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**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) P)))$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{3}$$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{4}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}) P) P)))$

**Definition 6** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2ESND ty\_2Efrac\_2Efrac))$ .  
Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod A\_27a A\_27b)} \end{aligned} \quad (6)$$

**Definition 7** We define  $c\_2Efrac\_2Efrac\_nmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2EFST ty\_2Efrac\_2Efrac))$ .  
Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty ty\_2Enum\_2Enum \quad (7)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint\_REP\_CLASS}) \quad (8)$$

**Definition 8** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E40 ty\_2Einteger\_2Eint))$ .  
Let  $c\_2Einteger\_2Eint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_neg \in ((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)^{ty\_2Einteger\_2Eint\_neg}) \quad (9)$$

Let  $c\_2Einteger\_2Eint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint\_eq}) \quad (10)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)})} \quad (11)$$

**Definition 9** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Einteger\_2Eint)$ .

**Definition 10** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap c\_2Einteger\_2Eint)$ .

**Definition 11** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 12** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21 2) (\lambda V2t \in 2))))$ .

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (12)$$

**Definition 13** We define  $c\_2Epair\_2E2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epair\_2E2C))$ .

Let  $c\_2Efrac\_2Eabs\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Eabs\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \quad (13)$$

**Definition 14** We define  $c\_2Efrac\_2Efrac\_ainv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_frac)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (14)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (15)$$

**Definition 15** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 16** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (16)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (17)$$

**Definition 17** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (18)$$

**Definition 18** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ n))$

**Definition 19** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (19)$$

Let  $c\_2Einteger\_2Etint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (20)$$

**Definition 20** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger\_2Eint.(ap\ c\_2Einteger\_2Etint\_add\ T1\ T2)$

**Definition 21** We define  $c\_2Efrac\_2Efrac\_save$  to be  $\lambda V0nmr \in ty\_2Einteger\_2Eint.\lambda V1dnm \in ty\_2Enum\_2Enum.(ap\ c\_2Einteger\_2Eint\_add\ n\ m\ r\ d\ n\ m)$

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)} \quad (21)$$

**Definition 22** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$ .

**Definition 23** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 24** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$ .

**Definition 25** We define  $c\_2Einteger\_2Eint\_le$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2Eint$ .

Let  $c\_2Einteger\_2Etint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)} \quad (22)$$

**Definition 26** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (23)$$

**Definition 27** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$ .

**Definition 28** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$ .

**Definition 29** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$ .

**Definition 30** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V3t3 \in 2.V3t3))))))$ .

**Definition 31** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 32** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ c\_2Ebool\_2E\_21\ 2))$ .

Assume the following.

$$True \quad (24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee \neg(p\ V0t))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p\ V0t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee \\ & (p\ V1B)))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Einteger\_2Eint.(\forall V1b \in ty\_2Einteger\_2Eint. \\ & ((p\ (ap\ (ap\ c\_2Einteger\_2Eint\_lt\ (ap\ c\_2Einteger\_2Eint\_of\_num \\ & c\_2Enum\_2E0))\ V1b)) \Rightarrow ((ap\ c\_2Efrac\_2Efrac\_nmr\ (ap\ c\_2Efrac\_2Eabs\_frac \\ & (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint) \\ & V0a)\ V1b))) = V0a)))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty\_2Einteger\_2Eint. (\forall V1b \in ty\_2Einteger\_2Eint. \\
& ((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) V1b)) \Rightarrow ((ap c\_2Efrac\_2Efrac\_dnm (ap c\_2Efrac\_2Eabs\_frac \\
& (ap (ap (c\_2Epair\_2E\_2C ty\_2Einteger\_2Eint ty\_2Einteger\_2Eint) \\
& V0a) V1b))) = V1b))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\
& ((p (ap (ap c\_2Einteger\_2Eint\_le V0x) V1y)) \Leftrightarrow (p (ap (ap c\_2Einteger\_2Eint\_le \\
& (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Einteger\_2Eint\_add \\
& V1y) (ap c\_2Einteger\_2Eint\_neg V0x))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Einteger\_2Eint}). ((\forall V1n \in ty\_2Enum\_2Enum. \\
& (p (ap V0P (ap c\_2Einteger\_2Eint\_of\_num V1n)))) \Leftrightarrow (\forall V2x \in \\
& ty\_2Einteger\_2Eint. ((p (ap (ap c\_2Einteger\_2Eint\_le (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) V2x)) \Rightarrow (p (ap V0P V2x))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty\_2Einteger\_2Eint. (\forall V1x \in ty\_2Einteger\_2Eint. \\
& (\forall V2y \in ty\_2Einteger\_2Eint. ((p (ap (ap c\_2Einteger\_2Eint\_le \\
& (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Einteger\_2Eint\_add \\
& V0c) V1x))) \Rightarrow ((p (ap (ap c\_2Einteger\_2Eint\_lt V2y) (ap c\_2Einteger\_2Eint\_neg \\
& V1x))) \Rightarrow ((p (ap (ap c\_2Einteger\_2Eint\_le (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Einteger\_2Eint\_add (ap c\_2Einteger\_2Eint\_neg \\
& V0c) V2y))) \Leftrightarrow False))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty\_2Einteger\_2Eint. (\forall V1x \in ty\_2Einteger\_2Eint. \\
& ((ap (ap c\_2Einteger\_2Eint\_add V1x) V0y) = (ap (ap c\_2Einteger\_2Eint\_add \\
& V0y) V1x))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\
& (\forall V2x \in ty\_2Einteger\_2Eint. ((ap (ap c\_2Einteger\_2Eint\_add \\
& V2x) (ap (ap c\_2Einteger\_2Eint\_add V1y) V0z)) = (ap (ap c\_2Einteger\_2Eint\_add \\
& (ap (ap c\_2Einteger\_2Eint\_add V2x) V1y)) V0z))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint. ((ap (ap c\_2Einteger\_2Eint\_add \\
& V0x) (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) = V0x))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint. ((ap (ap c\_2Einteger\_2Eint\_mul \\
& (ap c\_2Einteger\_2Eint\_of\_num (ap c\_2Earithmic\_2ENUMERAL \\
& (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)))) V0x) = V0x)) \quad (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\
& ((ap c\_2Einteger\_2Eint\_neg (ap (ap c\_2Einteger\_2Eint\_add V0x) \\
& V1y)) = (ap (ap c\_2Einteger\_2Eint\_add (ap c\_2Einteger\_2Eint\_neg \\
& V0x)) (ap c\_2Einteger\_2Eint\_neg V1y)))))) \quad (46)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\
& ((ap c\_2Einteger\_2Eint\_neg (ap (ap c\_2Einteger\_2Eint\_mul V0x) \\
& V1y)) = (ap (ap c\_2Einteger\_2Eint\_mul (ap c\_2Einteger\_2Eint\_neg \\
& V0x)) V1y)))) \quad (47)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\
& ((ap c\_2Einteger\_2Eint\_neg (ap (ap c\_2Einteger\_2Eint\_mul V0x) \\
& V1y)) = (ap (ap c\_2Einteger\_2Eint\_mul V0x) (ap c\_2Einteger\_2Eint\_neg \\
& V1y)))))) \quad (48)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) \quad (49)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty\_2Einteger\_2Eint. (\forall V1n \in ty\_2Enum\_2Enum. \\
& (\forall V2m \in ty\_2Enum\_2Enum. (((ap (ap c\_2Einteger\_2Eint\_add \\
& (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) V0p) = V0p) \wedge ((( \\
& ap (ap c\_2Einteger\_2Eint\_add V0p) (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) = V0p) \wedge (((ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num \\
& c\_2Enum\_2E0)) = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) \wedge \\
& (((ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_neg V0p)) = \\
& V0p) \wedge (((ap (ap c\_2Einteger\_2Eint\_add (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL V1n))) (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL V2m))) = (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B \\
& V1n) V2m)))))) \wedge (((ap (ap c\_2Einteger\_2Eint\_add (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL V1n))) (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& V2m)))) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Einteger\_2Eint) (ap \\
& (ap c\_2Earithmetic\_2E\_3C\_3D V2m) V1n)) (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V1n) \\
& V2m)))) (ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V2m) \\
& V1n)))))) \wedge (((ap (ap c\_2Einteger\_2Eint\_add (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& V1n))) (ap c\_2Einteger\_2Eint\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& V2m))) = (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Einteger\_2Eint) (ap ( \\
& ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2m)) (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V2m) \\
& V1n)))) (ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D V1n) \\
& V2m)))))) \wedge (((ap (ap c\_2Einteger\_2Eint\_add (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& V1n))) (ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL V2m)))) = (ap c\_2Einteger\_2Eint\_neg \\
& (ap c\_2Einteger\_2Eint\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2m))))))))))))))
\end{aligned}$$

(50)



Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& \quad ((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad c\_2Enum\_2E0)) \ (ap \ c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \\
& \quad (ap \ c\_2Earithmic\_2EBIT1 \ V0n)))))) \Leftrightarrow True) \wedge (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \\
& \quad (ap \ c\_2Integer\_2Eint\_of\_num \ c\_2Enum\_2E0)) \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad (ap \ c\_2Earithmic\_2ENUMERAL \ (ap \ c\_2Earithmic\_2EBIT2 \ V0n)))))) \Leftrightarrow \\
& \quad True) \wedge (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad c\_2Enum\_2E0)) \ (ap \ c\_2Integer\_2Eint\_of\_num \ c\_2Enum\_2E0))) \Leftrightarrow \\
& \quad False) \wedge (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad c\_2Enum\_2E0)) \ (ap \ c\_2Integer\_2Eint\_neg \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad (ap \ c\_2Earithmic\_2ENUMERAL \ V0n)))))) \Leftrightarrow False) \wedge (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \\
& \quad (ap \ c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \\
& \quad V0n)) \ (ap \ c\_2Integer\_2Eint\_of\_num \ c\_2Enum\_2E0))) \Leftrightarrow False) \wedge \\
& \quad (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \ (ap \ c\_2Integer\_2Eint\_neg \\
& \quad (ap \ c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \\
& \quad (ap \ c\_2Earithmic\_2EBIT1 \ V0n)))))) \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad c\_2Enum\_2E0))) \Leftrightarrow True) \wedge (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \ (ap \\
& \quad c\_2Integer\_2Eint\_neg \ (ap \ c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \\
& \quad (ap \ c\_2Earithmic\_2EBIT2 \ V0n)))))) \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad c\_2Enum\_2E0))) \Leftrightarrow True) \wedge (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \ (ap \\
& \quad c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \ V0n))) \\
& \quad (ap \ c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \\
& \quad V1m)))) \Leftrightarrow (p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ V0n \ V1m))) \wedge (((p \ (ap \ (ap \\
& \quad c\_2Integer\_2Eint\_lt \ (ap \ c\_2Integer\_2Eint\_neg \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad (ap \ c\_2Earithmic\_2ENUMERAL \ (ap \ c\_2Earithmic\_2EBIT1 \ V0n)))))) \\
& \quad (ap \ c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \\
& \quad V1m)))) \Leftrightarrow True) \wedge (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \ (ap \ c\_2Integer\_2Eint\_neg \\
& \quad (ap \ c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \\
& \quad (ap \ c\_2Earithmic\_2EBIT2 \ V0n)))))) \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad (ap \ c\_2Earithmic\_2ENUMERAL \ V1m)))) \Leftrightarrow True) \wedge (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \\
& \quad (ap \ c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \\
& \quad V0n)) \ (ap \ c\_2Integer\_2Eint\_neg \ (ap \ c\_2Integer\_2Eint\_of\_num \\
& \quad (ap \ c\_2Earithmic\_2ENUMERAL \ V1m)))))) \Leftrightarrow False) \wedge (((p \ (ap \ (ap \ c\_2Integer\_2Eint\_lt \\
& \quad (ap \ c\_2Integer\_2Eint\_neg \ (ap \ c\_2Integer\_2Eint\_of\_num \ ( \\
& \quad ap \ c\_2Earithmic\_2ENUMERAL \ V0n)))) \ (ap \ c\_2Integer\_2Eint\_neg \\
& \quad (ap \ c\_2Integer\_2Eint\_of\_num \ (ap \ c\_2Earithmic\_2ENUMERAL \\
& \quad V1m)))))) \Leftrightarrow (p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ V1m \ V0n))))))))))))) \\
& \hspace{10em} (51)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2. ((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (60)$$

### Theorem 1

$$(\forall V0a1 \in ty\_2Einteger\_2Eint. (\forall V1b1 \in ty\_2Enum\_2Enum. ((ap c\_2Efrac\_2Efrac\_ainv (ap (ap c\_2Efrac\_2Efrac\_save V0a1) V1b1)) = (ap (ap c\_2Efrac\_2Efrac\_save (ap c\_2Einteger\_2Eint\_neg V0a1)) V1b1))))$$