

thm_2Efrac_2EFRAC__MUL__SAVE (TM- MXResJYMtVQ6CXZZr7bVAb3C1xZ3iCgwq)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a$

Let `ty_2Einteger_2Eint` : ι be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \tag{1}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let `ty_2Efrac_2Efrac` : ι be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \tag{3}$$

Let `c_2Efrac_2Erep_frac` : ι be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \tag{4}$$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \tag{5}$$

Definition 4 We define `c_2Ebool_2ET` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}$

Definition 6 We define $c_2Efrac_2Efrac_dnm$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2ESND ty$
 Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (6)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{ty_2Einteger_2Eint}) \quad (7)$$

Definition 7 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E.40 (ty$

Let $c_2Einteger_2Eint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_mul \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (8)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (9)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})} \quad (10)$$

Definition 8 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 9 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a\ A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (11)$$

Definition 10 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap (c_2Epair_2EFST ty$

Definition 11 We define $c_2Emin_2E.3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$
 of type ι .

Definition 12 We define $c_2Ebool_2E.2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E.21\ 2) (\lambda V2t \in 2.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b} A_27a})} \quad (12)$$

Definition 13 We define $c_2Epair_2E.2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Efrac_2Eabs_frac : \iota$ be given. Assume the following.

$$c_2Efrac_2Eabs_frac \in (ty_2Efrac_2Efrac^{(ty_2Epair_2Eprod\ ty_2Einteger_2Eint\ ty_2Einteger_2Eint)}) \quad (13)$$

Definition 14 We define $c_2Efrac_2Efrac_mul$ to be $\lambda V0f1 \in ty_2Efrac_2Efrac.\lambda V1f2 \in ty_2Efrac_2Efrac$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 16 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 17 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 18 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ n))$.

Definition 19 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Einteger_2Etint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (20)$$

Definition 20 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$.

Definition 21 We define $c_2Efrac_2Efrac_save$ to be $\lambda V0nmr \in ty_2Einteger_2Eint.\lambda V1dnm \in ty_2Enum$.

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})$$

(21)

Definition 22 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$.

Definition 23 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 24 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$.

Definition 25 We define $c_2Einteger_2Eint_le$ to be $\lambda V0x \in ty_2Einteger_2Eint.\lambda V1y \in ty_2Einteger_2Eint$.

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$

(22)

Definition 26 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Definition 27 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$.

Definition 28 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.V2t2)))$.

Definition 30 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$$

(23)

Definition 31 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2A\ V0n))$.

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum})$$

(24)

Definition 32 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint_neg\ T1)$.

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B\ V1n)\ V0m))))$$

(25)

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
& (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (\\
& ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& \quad V0m) = V0m))
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V0m) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
& \quad V0m))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap \\
& (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p) = (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A V0m) V2p)) (ap (ap c_2Earithmetic_2E_2A \\
& \quad V1n) V2p))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V2p) \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) = (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2A V2p) V0m)) (ap (ap c_2Earithmetic_2E_2A \\
& \quad V2p) V1n))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2B V0m) \\
& V2p) = (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) \Leftrightarrow (V0m = V1n))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\text{True} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))))
\end{aligned} \tag{33}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_27a. (p V0t)) \Leftrightarrow (p V0t))) \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge \\ & ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p V0t)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p (ap V0P V2x)))))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (44)$$

Assume the following.

$$(\forall V0a \in ty_2Einteger_2Eint.(\forall V1b \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V1b)) \Rightarrow ((ap c_2Efrac_2Efrac_nmr (ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) V0a) V1b))) = V0a)))))) \quad (45)$$

Assume the following.

$$(\forall V0a \in ty_2Einteger_2Eint.(\forall V1b \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V1b)) \Rightarrow ((ap c_2Efrac_2Efrac_dnm (ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) V0a) V1b))) = V1b)))))) \quad (46)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_le V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V1y) (ap c_2Einteger_2Eint_neg V0x)))))))) \quad (47)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Einteger_2Eint}).(\forall V1n \in ty_2Enum_2Enum. (p (ap V0P (ap c_2Einteger_2Eint_of_num V1n)))) \Leftrightarrow (\forall V2x \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V2x)) \Rightarrow (p (ap V0P V2x)))))) \quad (48)$$

Assume the following.

$$(\forall V0c \in ty_2Einteger_2Eint.(\forall V1x \in ty_2Einteger_2Eint. (\forall V2y \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V0c) V1x))) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_lt V2y) (ap c_2Einteger_2Eint_neg V1x))) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg V0c) V2y)))) \Leftrightarrow False)))))) \quad (49)$$

Assume the following.

$$(\forall V0y \in ty_2Einteger_2Eint.(\forall V1x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add V1x) V0y) = (ap (ap c_2Einteger_2Eint_add V0y) V1x)))) \quad (50)$$

Assume the following.

$$\begin{aligned}
& (\forall V0z \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& (\forall V2x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add \\
V2x) (ap (ap c_2Einteger_2Eint_add V1y) V0z)) = (ap (ap c_2Einteger_2Eint_add \\
& (ap (ap c_2Einteger_2Eint_add V2x) V1y)) V0z))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add \\
V0x) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_add V0x) \\
V1y)) = (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& V0x)) (ap c_2Einteger_2Eint_neg V1y))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) \\
V1y)) = (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_neg \\
& V0x)) V1y))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Einteger_2Eint. (\forall V1y \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) \\
V1y)) = (ap (ap c_2Einteger_2Eint_mul V0x) (ap c_2Einteger_2Eint_neg \\
& V1y))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& ((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2m \in ty_2Enum_2Enum. (((ap (ap c_2Einteger_2Eint_add \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0p) = V0p) \wedge (((\\
& ap (ap c_2Einteger_2Eint_add V0p) (ap c_2Einteger_2Eint_of_num \\
c_2Enum_2E0)) = V0p) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
V1n)) (ap c_2Einteger_2Eint_of_num V2m)) = (ap c_2Einteger_2Eint_of_num \\
(ap (ap c_2Earithmetic_2E_2B V1n) V2m))) \wedge (((ap (ap c_2Einteger_2Eint_add \\
(ap c_2Einteger_2Eint_of_num V1n)) (ap c_2Einteger_2Eint_neg \\
(ap c_2Einteger_2Eint_of_num V2m))) = (ap (ap (ap (c_2Ebool_2ECOND \\
ty_2Einteger_2Eint) (ap (ap c_2Earithmetic_2E_3C_3D V2m) V1n)) \\
(ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2D \\
V1n) V2m))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
(ap (ap c_2Earithmetic_2E_2D V2m) V1n)))))) \wedge (((ap (ap c_2Einteger_2Eint_add \\
(ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V1n)) \\
(ap c_2Einteger_2Eint_of_num V2m)) = (ap (ap (ap (c_2Ebool_2ECOND \\
ty_2Einteger_2Eint) (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2m)) \\
(ap c_2Einteger_2Eint_of_num (ap (ap c_2Earithmetic_2E_2D \\
V2m) V1n))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
(ap (ap c_2Earithmetic_2E_2D V1n) V2m)))))) \wedge ((ap (ap c_2Einteger_2Eint_add \\
(ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V1n)) \\
(ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num V2m))) = \\
(ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num (\\
ap (ap c_2Earithmetic_2E_2B V1n) V2m)))))))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2m \in ty_2Enum_2Enum. (((ap (ap c_2Einteger_2Eint_add \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0p) = V0p) \wedge (((\\
& ap (ap c_2Einteger_2Eint_add V0p) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = V0p) \wedge (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \wedge \\
& (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V0p)) = \\
& V0p) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m))) = (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\
& V1n) V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m)))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap \\
& (ap c_2Earithmetic_2E_3C_3D V2m) V1n)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap (\\
& ap c_2Earithmetic_2E_3C_3D V1n) V2m)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m)))) = (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B V1n) V2m))))))))))))))
\end{aligned}$$

(59)

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\
V0m)) (ap c_2Einteger_2Eint_of_num V1n)) = (ap c_2Einteger_2Eint_of_num \\
& (ap (ap c_2Earithmic_2E_2A V0m) V1n)))))) \wedge ((\forall V2x \in ty_2Einteger_2Eint. \\
& (\forall V3y \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& (ap c_2Einteger_2Eint_neg V2x)) V3y) = (ap c_2Einteger_2Eint_neg \\
& (ap (ap c_2Einteger_2Eint_mul V2x) V3y)))))) \wedge ((\forall V4x \in ty_2Einteger_2Eint. \\
& (\forall V5y \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\
& V4x) (ap c_2Einteger_2Eint_neg V5y)) = (ap c_2Einteger_2Eint_neg \\
& (ap (ap c_2Einteger_2Eint_mul V4x) V5y)))))) \wedge ((\forall V6x \in ty_2Einteger_2Eint. \\
& ((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V6x)) = \\
& V6x))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (((ap\ c_2Einteger_2Eint_of_num\ V0m) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad V1n)) \Leftrightarrow (V0m = V1n)))) \wedge ((\forall V2x \in ty_2Einteger_2Eint. (\forall V3y \in \\
& ty_2Einteger_2Eint. (((ap\ c_2Einteger_2Eint_neg\ V2x) = (ap\ c_2Einteger_2Eint_neg \\
& \quad V3y)) \Leftrightarrow (V2x = V3y)))) \wedge (\forall V4n \in ty_2Enum_2Enum. (\forall V5m \in \\
& ty_2Enum_2Enum. (((ap\ c_2Einteger_2Eint_of_num\ V4n) = (ap \\
& \quad c_2Einteger_2Eint_neg\ (ap\ c_2Einteger_2Eint_of_num\ V5m))) \Leftrightarrow \\
& ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))) \wedge (((ap\ c_2Einteger_2Eint_neg \\
& \quad (ap\ c_2Einteger_2Eint_of_num\ V4n)) = (ap\ c_2Einteger_2Eint_of_num \\
& \quad V5m)) \Leftrightarrow ((V4n = c_2Enum_2E0) \wedge (V5m = c_2Enum_2E0))))))))) \\
& \tag{62}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{63}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{64}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \tag{65}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \tag{66}
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{67}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))))) \\
& \tag{68}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\
& (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\
& (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \\
& \tag{69}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{71}$$

Theorem 1

$$\begin{aligned}
& (\forall V0a1 \in ty_2Einteger_2Eint. (\forall V1b1 \in ty_2Enum_2Enum. \\
& (\forall V2a2 \in ty_2Einteger_2Eint. (\forall V3b2 \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Efrac_2Efrac_mul (ap (ap c_2Efrac_2Efrac_save V0a1) \\
V1b1)) (ap (ap c_2Efrac_2Efrac_save V2a2) V3b2)) = (ap (ap c_2Efrac_2Efrac_save \\
& (ap (ap c_2Einteger_2Eint_mul V0a1) V2a2)) (ap (ap c_2Earithmetic_2E_2B \\
& (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V1b1) \\
V3b2)) V1b1)) V3b2))))))
\end{aligned}$$