

thm_2Efrac_2EFRAC__NMR__SAVE
 (TMWcmDtBfVN-
 WMBR3UjWN2KAB9MhEgyUtoVu)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 4 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 5 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 6 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (7)$$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (8)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (9)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint}) \quad (10)$$

Definition 11 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint. (ap (c_2Emin_2E_40 (ty_2Einteger_2Eint_REP_CLASS$

Let $c_2Einteger_2Eint_add : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_add \in (((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (11)$$

Let $c_2Einteger_2Eint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)}) \quad (12)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})}) \quad (13)$$

Definition 12 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)$

Definition 13 We define $c_2Einteger_2Eint_add$ to be $\lambda V0T1 \in ty_2Einteger_2Eint. \lambda V1T2 \in ty_2Einteger_2Eint$

Definition 14 We define $c_Emin_E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 15 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21 2) (\lambda V2t \in 2.))$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EABS_prod A_27a A_27b \in ((ty_Epair_Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}})$$
(14)

Definition 16 We define $c_Epair_E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_Epair_EABS_prod$

Let $ty_Efrac_Efrac : \iota$ be given. Assume the following.

$$nonempty ty_Efrac_Efrac$$
(15)

Let $c_Efrac_Eabs_frac : \iota$ be given. Assume the following.

$$c_Efrac_Eabs_frac \in (ty_Efrac_Efrac^{(ty_Epair_Eprod ty_Einteger_Eint ty_Einteger_Eint)})$$
(16)

Definition 17 We define $c_Efrac_Efrac_save$ to be $\lambda V0nmr \in ty_Einteger_Eint.\lambda V1dnm \in ty_Eenum_Eenum$

Let $c_Efrac_Erep_frac : \iota$ be given. Assume the following.

$$c_Efrac_Erep_frac \in ((ty_Epair_Eprod ty_Einteger_Eint ty_Einteger_Eint)^{ty_Efrac_Efrac})$$
(17)

Let $c_Epair_E_EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_E_EFST A_27a A_27b \in (A_27a^{(ty_Epair_Eprod A_27a A_27b)})$$
(18)

Definition 18 We define $c_Efrac_Efrac_nmr$ to be $\lambda V0f \in ty_Efrac_Efrac.(ap (c_Epair_E_EFST ty_Einteger_Eint ty_Einteger_Eint$

Let $c_Einteger_E_Etint_lt : \iota$ be given. Assume the following.

$$c_Einteger_E_Etint_lt \in ((2^{(ty_Epair_Eprod ty_Eenum_Eenum ty_Eenum_Eenum)})^{(ty_Epair_Eprod ty_Eenum_Eenum)})$$
(19)

Definition 19 We define $c_Einteger_E_Eint_lt$ to be $\lambda V0T1 \in ty_Einteger_Eint.\lambda V1T2 \in ty_Einteger_Eint$

Definition 20 We define $c_Ebool_E_EF$ to be $(ap (c_Ebool_E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 21 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_E_3D_3D_3E V0t) c_Ebool_E_21 2))$

Definition 22 We define $c_Einteger_E_Eint_le$ to be $\lambda V0x \in ty_Einteger_Eint.\lambda V1y \in ty_Einteger_Eint$

Let $c_2Einteger_2Etint_mul : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_mul \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \quad (20)$$

Definition 23 We define $c_2Einteger_2Eint_mul$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger_2Eint$.
Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (21)$$

Definition 24 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 25 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.$

Definition 26 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Definition 27 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 28 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})_{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)} \quad (22)$$

Definition 29 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap\ c_2Einteger_2Eint$.

Definition 30 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p \ V0t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \vee (p \ V1B)) \Leftrightarrow ((p \ V1B) \vee (p \ V0A)))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (38)$$

Assume the following.

$$(\forall V0a \in ty_2Einteger_2Eint.(\forall V1b \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V1b)) \Rightarrow ((ap c_2Efrac_2Efrac_nmr (ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2E_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) V0a) V1b))) = V0a)))))) \quad (39)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. ((p (ap (ap c_2Einteger_2Eint_le V0x) V1y)) \Leftrightarrow (p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V1y) (ap c_2Einteger_2Eint_neg V0x))))))) \quad (40)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Einteger_2Eint}).((\forall V1n \in ty_2Enum_2Enum. (p (ap V0P (ap c_2Einteger_2Eint_of_num V1n)))) \Leftrightarrow (\forall V2x \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V2x)) \Rightarrow (p (ap V0P V2x))))))) \quad (41)$$

Assume the following.

$$(\forall V0c \in ty_2Einteger_2Eint.(\forall V1x \in ty_2Einteger_2Eint. (\forall V2y \in ty_2Einteger_2Eint.((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add V0c) V1x))) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_lt V2y) (ap c_2Einteger_2Eint_neg V1x))) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_le (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg V0c) V2y)))) \Leftrightarrow False)))))) \quad (42)$$

Assume the following.

$$(\forall V0y \in ty_2Einteger_2Eint.(\forall V1x \in ty_2Einteger_2Eint. ((ap (ap c_2Einteger_2Eint_add V1x) V0y) = (ap (ap c_2Einteger_2Eint_add V0y) V1x)))) \quad (43)$$

Assume the following.

$$(\forall V0z \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. (\forall V2x \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_add V2x) (ap (ap c_2Einteger_2Eint_add V1y) V0z)) = (ap (ap c_2Einteger_2Eint_add (ap (ap c_2Einteger_2Eint_add V2x) V1y)) V0z)))))) \quad (44)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_add V0x) (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = V0x)) \quad (45)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))) V0x) = V0x)) \quad (46)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_add V0x) V1y)) = (ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg V0x)) (ap c_2Einteger_2Eint_neg V1y)))))) \quad (47)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) V1y)) = (ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_neg V0x)) V1y)))) \quad (48)$$

Assume the following.

$$(\forall V0x \in ty_2Einteger_2Eint.(\forall V1y \in ty_2Einteger_2Eint. ((ap c_2Einteger_2Eint_neg (ap (ap c_2Einteger_2Eint_mul V0x) V1y)) = (ap (ap c_2Einteger_2Eint_mul V0x) (ap c_2Einteger_2Eint_neg V1y)))))) \quad (49)$$

Assume the following.

$$((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \quad (50)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in ty_2Einteger_2Eint. (\forall V1n \in ty_2Enum_2Enum. \\
& (\forall V2m \in ty_2Enum_2Enum. (((ap (ap c_2Einteger_2Eint_add \\
& (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) V0p) = V0p) \wedge (((\\
& ap (ap c_2Einteger_2Eint_add V0p) (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = V0p) \wedge (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& c_2Enum_2E0)) = (ap c_2Einteger_2Eint_of_num c_2Enum_2E0)) \wedge \\
& (((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V0p)) = \\
& V0p) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m))) = (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B \\
& V1n) V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V1n))) (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m)))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap \\
& (ap c_2Earithmetic_2E_3C_3D V2m) V1n)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V2m))) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Einteger_2Eint) (ap (\\
& ap c_2Earithmetic_2E_3C_3D V1n) V2m)) (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V2m) \\
& V1n)))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V1n) \\
& V2m)))))) \wedge (((ap (ap c_2Einteger_2Eint_add (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& V1n))) (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\
& (ap c_2Earithmetic_2ENUMERAL V2m)))) = (ap c_2Einteger_2Eint_neg \\
& (ap c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B V1n) V2m))))))))))))))
\end{aligned}$$

(51)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& \quad ((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_of_num \\
& \quad c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& \quad (ap c_2Earithmic_2EBIT1 V0n)))))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& (ap c_2Integer_2Eint_of_num c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT2 V0n)))))) \Leftrightarrow \\
& \quad True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_of_num \\
& c_2Enum_2E0)) (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow \\
& \quad False) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_of_num \\
& c_2Enum_2E0)) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& (ap c_2Earithmic_2ENUMERAL V0n)))))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& (ap c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& V0n))) (ap c_2Integer_2Eint_of_num c_2Enum_2E0))) \Leftrightarrow False) \wedge \\
& \quad (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_neg \\
& (ap c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT1 V0n)))))) (ap c_2Integer_2Eint_of_num \\
& c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap \\
& c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT2 V0n)))))) (ap c_2Integer_2Eint_of_num \\
& c_2Enum_2E0))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap \\
& c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL V0n))) \\
& (ap c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& V1m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0n V1m))) \wedge (((p (ap (ap \\
& c_2Integer_2Eint_lt (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 V0n)))))) \\
& (ap c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& V1m)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt (ap c_2Integer_2Eint_neg \\
& (ap c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT2 V0n)))))) (ap c_2Integer_2Eint_of_num \\
& (ap c_2Earithmic_2ENUMERAL V1m)))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& (ap c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& V0n))) (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num \\
& (ap c_2Earithmic_2ENUMERAL V1m)))))) \Leftrightarrow False) \wedge (((p (ap (ap c_2Integer_2Eint_lt \\
& (ap c_2Integer_2Eint_neg (ap c_2Integer_2Eint_of_num (\\
& ap c_2Earithmic_2ENUMERAL V0n)))) (ap c_2Integer_2Eint_neg \\
& (ap c_2Integer_2Eint_of_num (ap c_2Earithmic_2ENUMERAL \\
& V1m)))))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n))))))))))))) \\
& \hspace{10em} (52)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (54)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r))) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (62)$$

Theorem 1

$$(\forall V0a1 \in ty_2Einteger_2Eint. (\forall V1b1 \in ty_2Enum_2Enum. ((ap\ c_2Efrac_2Efrac_nmr (ap (ap\ c_2Efrac_2Efrac_save V0a1) V1b1)) = V0a1)))$$