

thm_2Efrac_2EFRAC__SGN__CASES (TMGLsg331r3kJMiw6ymZhchKTPhvKxDioVy)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o(x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_Ebool_2ET to be $(ap \ (ap \ (c_Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2.V0x)) \ (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

nonempty ty_2Enum_2Enum (2)

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (3)$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 4 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^\omega) \quad (5)$$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^A_{27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^A_{27a}\ P)\ V)\ 0)\ P))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ ($

Let c_2 be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 n) 0)$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Einteger_2Eint : \iota$ be given. Assume the following.

$$nonempty\ ty_2Einteger_2Eint \quad (7)$$

Let $c_2Einteger_2Eint_of_num : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_of_num \in (ty_2Einteger_2Eint^{ty_2Enum_2Enum}) \quad (8)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ A0\ A1) \end{aligned} \quad (9)$$

Let $c_2Einteger_2Eint_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{ty_2Einteger_2Eint})^{ty_2Einteger_2Eint} \quad (10)$$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\text{the } (\lambda x.x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

Definition 10 We define $c_2Einteger_2Eint_REP$ to be $\lambda V0a \in ty_2Einteger_2Eint.(ap (c_2Emin_2E_40 (ty_2Einteger_2Eint a)))$

Let $c_2Einteger_2Etint_neg : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_neg \in ((ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)} \quad (11)$$

Let $c_2Einteger_2Etint_eq : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)} \quad (12)$$

Let $c_2Einteger_2Eint_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Einteger_2Eint_ABS_CLASS \in (ty_2Einteger_2Eint)^{(2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}} \quad (13)$$

Definition 11 We define $c_2Einteger_2Eint_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum).ap (c_2Einteger_2Eint_ABS r)$

Definition 12 We define $c_2Einteger_2Eint_neg$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.(ap c_2Einteger_2Eint_neg T1)$

Let $ty_2Efrac_2Efrac : \iota$ be given. Assume the following.

$$nonempty\ ty_2Efrac_2Efrac \quad (14)$$

Let $c_2 \in \mathbb{F}$ be given. Assume the following.

$$c_2Efrac_2Erep_frac \in ((ty_2Epair_2Eprod\ ty_2Einteger_2Eint \\ ty_2Einteger_2Eint)^{ty_2Efrac_2Efrac}) \quad (15)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2\text{Epair_2EFST } A_27a \ A_27b \in (A_27a^{(ty_2\text{Epair_2Eprod } A_27a \ A_27b)})$$

Definition 13 We define $c_2Efrac_2Efrac_nmr$ to be $\lambda V0f \in ty_2Efrac_2Efrac.(ap(c_2Epair_2EFST\ ty_1$

Let $c_2Einteger_2Etint_lt : \iota$ be given. Assume the following.

$$c_2Einteger_2Etint_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum\ ty_2Enum_2Enum\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)} \quad (17)$$

Definition 14 We define $c_2Einteger_2Eint_lt$ to be $\lambda V0T1 \in ty_2Einteger_2Eint.\lambda V1T2 \in ty_2Einteger$

Definition 15 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t\in 2.V0t))$.

Definition 16 We define $c_{\text{min}} : \lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 17 We define $c_{\text{C_Ebool_2E_2F_5C}}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_{\text{C_Ebool_2E_21}}) 2))(\lambda V2t \in$

Definition 19 We define $c_2EintExtension_2ESGN$ to be $\lambda V0x \in ty_2Einteger_2Eint.(ap (ap (ap (ap (c_2Eboc$

Definition 20 We define $c_2 \in \text{Efrac_sgn}$ to be $\lambda V0 f1 \in \text{ty_Efrac_frac_ap_c2_EintExtension_ES}$

Definition 21 We define $c \in \text{Ebool}$ to be $(\lambda V_0 t_1 \in 2, (\lambda V_1 t_2 \in 2, (ap(c, \lambda V_0 t_1 \in 2, \lambda V_1 t_2 \in 2) \rightarrow (\lambda V_2 t_3 \in 2, \lambda V_3 t_4 \in 2)))$

Definition 22 We define $c \in \text{Ebool} \cap \text{E}(\text{Emin}, \text{E3D}, \text{E3D}, \text{E3E}, V0t)$ to be $(\lambda V0t \in \text{E}(\text{ap}, (\text{ap} \circ \text{Emin}), \text{E3D}, \text{E3D}, \text{E3E}), V0t) \in \text{Ebool} \cap \text{E}(\text{Emin}, \text{E3D}, \text{E3D}, \text{E3E})$.

Assume the following

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$$V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2))))))) \quad (19)$$

$$((x+1)^{-1} - ((x+1)^{-1}) \cdot ((x+1)^{-1}))^{-1}$$

Please see the following:

$$(\forall \alpha \in \Sigma. (((p \vee \alpha) , \text{false}) , ((p \vee \alpha)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & \quad True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & \quad (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & \quad ((\neg False) \Leftrightarrow True)))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & \quad (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & \quad (p V0t))))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B)) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \wedge \\ & \quad ((\neg(p V0A) \vee (\neg(p V1B))) \wedge ((\neg((p V0A) \vee (p V1B)) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0f1 \in ty_2Efrac_2Efrac.(((ap c_2Efrac_2Efrac_sgn \\ & \quad V0f1) = (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\ & \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \vee \\ & \quad (((ap c_2Efrac_2Efrac_sgn V0f1) = (ap c_2Einteger_2Eint_of_num \\ & \quad c_2Enum_2E0)) \vee ((ap c_2Efrac_2Efrac_sgn V0f1) = (ap c_2Einteger_2Eint_of_num \\ & \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \end{aligned} \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A) \Rightarrow False))) \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B)) \Rightarrow False) \Leftrightarrow \\ & \quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B)) \Rightarrow False) \Leftrightarrow \\ & \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & ((\neg(p V0p)) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee ((\neg(p V2r)) \wedge \\ & ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (37)$$

Theorem 1

$$\begin{aligned} & (\forall V0f1 \in ty_2Efrac_2Efrac.(\forall V1P \in 2.((((ap c_2Efrac_2Efrac_sgn \\ V0f1) = (ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_of_num \\ (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \Rightarrow \\ & (p V1P)) \wedge (((ap c_2Efrac_2Efrac_sgn V0f1) = (ap c_2Einteger_2Eint_of_num \\ c_2Enum_2E0)) \Rightarrow (p V1P)) \wedge (((ap c_2Efrac_2Efrac_sgn V0f1) = (ap \\ c_2Einteger_2Eint_of_num (ap c_2Earithmetic_2ENUMERAL (ap \\ c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \Rightarrow (p V1P)))) \Rightarrow \\ & (p V1P))) \end{aligned}$$