

thm\_2Efrac\_2Efrac\_\_SGN\_\_NEG  
(TMNpN7FmtLbMfZFP4cV5tPC2TkacG5Vpix6)

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**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \tag{3}$$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \tag{4}$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c\_2Epair\_2EFST\ A.27a\ A.27b \in (A.27a^{(ty\_2Epair\_2Eprod\ A.27a\ A.27b)}) \tag{5}$$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap (ap (c\_2Emin\_2E3D (2^{A.27a}))$

**Definition 4** We define  $c\_2Efrac\_2Efrac\_nmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2EFST\ ty\_2Efrac\_2Efrac$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{6}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (7)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (8)$$

**Definition 5** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 6** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (omega^{omega}) \quad (10)$$

**Definition 7** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

Let  $c\_2Earithmic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 8** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmic\_2E\_2B\ ($

**Definition 9** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{ty\_2Einteger\_2Eint}) \quad (13)$$

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge P\ x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 11** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap\ (c\_2Emin\_2E\_40\ ($

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (14)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)}) \quad (15)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})}) \quad (16)$$

**Definition 12** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)$

**Definition 13** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap\ c\_2Einteger\_2Eint)$

Let  $c\_2Einteger\_2Eint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (17)$$

**Definition 14** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 15** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 16** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 17** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V0t))))$

**Definition 18** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t2 \in 2.V0t))))))$

**Definition 19** We define  $c\_2EintExtension\_2ESGN$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.(ap\ (ap\ (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))))$

**Definition 20** We define  $c\_2Efrac\_2Efrac\_sgn$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2EintExtension\_2ESGN)$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (18)$$

**Definition 21** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap\ (c\_2Epair\_2ESND\ t))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b} A\_27a)}) \end{aligned} \quad (19)$$

**Definition 22** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t)))$

Let  $c\_2Efrac\_2Eabs\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Eabs\_frac \in (ty\_2Efrac\_2Efrac^{(ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint\ ty\_2Einteger\_2Eint)}) \quad (20)$$

**Definition 23** We define  $c\_2Efrac\_2Efrac\_ainv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_frac)$

**Definition 24** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V0t))))$

**Definition 25** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_21\ 2))$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee \neg(p V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow \neg(p V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(\neg(p V0t) \Rightarrow ((p V0t) \Rightarrow False))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (29)$$

Assume the following.

$$((\forall V0t \in 2.(\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (31)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (33)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B) \vee (p\ V2C)) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A)))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))) \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1b \in 2. (\forall V2x \in A\_27a. (\forall V3y \in A\_27a. ((ap\ V0f\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1b)\ V2x)\ V3y)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27b)\ V1b)\ (ap\ V0f\ V2x))\ (ap\ V0f\ V3y)))))) \quad (39)$$

Assume the following.

$$(\forall V0b \in 2. (\forall V1t1 \in 2. (\forall V2t2 \in 2. ((p\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ 2)\ V0b)\ V1t1)\ V2t2)) \Leftrightarrow (((\neg(p\ V0b)) \vee (p\ V1t1)) \wedge ((p\ V0b) \vee (p\ V2t2))))) \quad (40)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (41)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}.(\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a}) V0P) V2x) V4y) = (ap (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a}) V1Q) V3x_{.27}) V5y_{.27})))))) \Rightarrow (42)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in A_{.27a}.((ap (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a}) c_{.2E}bool_{.2E}T) V0t1) V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap (ap (ap (c_{.2E}bool_{.2E}COND A_{.27a}) c_{.2E}bool_{.2E}F) V2t1) V3t2) = V3t2)))) \Rightarrow (43)$$

Assume the following.

$$(\forall V0f \in ty_{.2E}frac_{.2E}frac.((ap c_{.2E}frac_{.2E}abs_{.2E}frac (ap (ap (c_{.2E}pair_{.2E}.2C ty_{.2E}integer_{.2E}int ty_{.2E}integer_{.2E}int) (ap c_{.2E}frac_{.2E}frac_{.2E}nmr V0f)) (ap c_{.2E}frac_{.2E}frac_{.2E}dnm V0f))) = V0f)) \Rightarrow (44)$$

Assume the following.

$$(\forall V0f \in ty_{.2E}frac_{.2E}frac.(p (ap (ap c_{.2E}integer_{.2E}int_{.2E}lt (ap c_{.2E}integer_{.2E}int_{.2E}of_{.2E}num c_{.2E}enum_{.2E}E0) (ap c_{.2E}frac_{.2E}frac_{.2E}dnm V0f)))) \Rightarrow (45)$$

Assume the following.

$$(\forall V0a \in ty_{.2E}integer_{.2E}int.(\forall V1b \in ty_{.2E}integer_{.2E}int.((p (ap (ap c_{.2E}integer_{.2E}int_{.2E}lt (ap c_{.2E}integer_{.2E}int_{.2E}of_{.2E}num c_{.2E}enum_{.2E}E0) V1b)) \Rightarrow ((ap c_{.2E}frac_{.2E}frac_{.2E}nmr (ap c_{.2E}frac_{.2E}abs_{.2E}frac (ap (ap (c_{.2E}pair_{.2E}.2C ty_{.2E}integer_{.2E}int ty_{.2E}integer_{.2E}int) V0a) V1b))) = V0a)))) \Rightarrow (46)$$

Assume the following.

$$(\forall V0a \in ty_{.2E}integer_{.2E}int.(\forall V1b \in ty_{.2E}integer_{.2E}int.((p (ap (ap c_{.2E}integer_{.2E}int_{.2E}lt (ap c_{.2E}integer_{.2E}int_{.2E}of_{.2E}num c_{.2E}enum_{.2E}E0) V1b)) \Rightarrow ((ap c_{.2E}frac_{.2E}frac_{.2E}dnm (ap c_{.2E}frac_{.2E}abs_{.2E}frac (ap (ap (c_{.2E}pair_{.2E}.2C ty_{.2E}integer_{.2E}int ty_{.2E}integer_{.2E}int) V0a) V1b))) = V1b)))) \Rightarrow (47)$$

Assume the following.

$$(\forall V0n \in ty\_2Einteger\_2Eint.((\neg(p (ap (ap c\_2Einteger\_2Eint\_lt V0n) (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)))) \Leftrightarrow (((ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0) = V0n) \vee (p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) V0n)))))) \quad (48)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_neg V0x)) = V0x)) \quad (49)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.((p (ap (ap c\_2Einteger\_2Eint\_lt V0x) V1y)) \Rightarrow (\neg(p (ap (ap c\_2Einteger\_2Eint\_lt V1y) V0x))))) \quad (50)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.((p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_neg V0x) (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0))) \Leftrightarrow (p (ap (ap c\_2Einteger\_2Eint\_lt (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) V0x)))) \quad (51)$$

Assume the following.

$$((ap c\_2Einteger\_2Eint\_neg (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) = (ap c\_2Einteger\_2Eint\_of\_num c\_2Enum\_2E0)) \quad (52)$$

Assume the following.

$$(\forall V0x \in ty\_2Einteger\_2Eint.(\forall V1y \in ty\_2Einteger\_2Eint.(((ap c\_2Einteger\_2Eint\_neg V0x) = V1y) \Leftrightarrow (V0x = (ap c\_2Einteger\_2Eint\_neg V1y)))))) \quad (53)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap c\_2Einteger\_2Eint\_of\_num V0m) = (ap c\_2Einteger\_2Eint\_of\_num V1n)) \Leftrightarrow (V0m = V1n)))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (56)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (57)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r))) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (64)$$

### Theorem 1

$$(\forall V0f \in ty\_2Efrac\_2Efrac. ((ap\_c\_2Einteger\_2Eint\_neg (ap\_c\_2Efrac\_2Efrac\_sgn (ap\_c\_2Efrac\_2Efrac\_ainv V0f))) = (ap\_c\_2Efrac\_2Efrac\_sgn V0f)))$$