

# thm\_2Efrac\_2EFRAC\_\_SUB\_\_CALCULATE (TMHF92jU1M8i7U9GryKTpZErpQf3cBD38Kz)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Einteger\_2Eint : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Einteger\_2Eint \quad (1)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod \\ A0\ A1) \end{aligned} \quad (2)$$

Let  $ty\_2Efrac\_2Efrac : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Efrac\_2Efrac \quad (3)$$

Let  $c\_2Efrac\_2Erep\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Erep\_frac \in ((ty\_2Epair\_2Eprod\ ty\_2Einteger\_2Eint \\ ty\_2Einteger\_2Eint)^{ty\_2Efrac\_2Efrac}) \quad (4)$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (5)$$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Efrac\_2Efrac\_dnm$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac. (ap (c\_2Epair\_2ESND\ ty0f))$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (6)$$

**Definition 5** We define  $c\_2Efrac\_2Efrac\_nmr$  to be  $\lambda V0f \in ty\_2Efrac\_2Efrac.(ap (c\_2Epair\_2EFST ty\_2Efrac\_2Efrac\_nmr) f)$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \quad (7)$$

Let  $c\_2Einteger\_2Eint\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}) ty\_2Einteger\_2Eint) \quad (8)$$

**Definition 6** We define  $c\_2Emin\_2E_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \text{ } x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota))$

**Definition 7** We define  $c\_2Einteger\_2Eint\_REP$  to be  $\lambda V0a \in ty\_2Einteger\_2Eint.(ap (c\_2Emin\_2E_40 (ty\_2Einteger\_2Eint)))$

Let  $c\_2Einteger\_2Etint\_neg : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_neg \in ((ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum) ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)} \quad (9)$$

Let  $c\_2Einteger\_2Etint\_eq : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}) ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum)} \quad (10)$$

Let  $c\_2Einteger\_2Eint\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_ABS\_CLASS \in (ty\_2Einteger\_2Eint)^{(2^{(ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum)}) ty\_2Epair\_2Eprod ty\_2Enum\_2Enum} \quad (11)$$

**Definition 8** We define  $c\_2Einteger\_2Eint\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum) ty\_2Epair\_2Eprod ty\_2Enum\_2Enum ty\_2Enum\_2Enum$

**Definition 9** We define  $c\_2Einteger\_2Eint\_neg$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.(ap c\_2Einteger\_2Eint\_neg)$

**Definition 10** We define  $c\_2Emin\_2E_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p P \Rightarrow p Q)$   
of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E_21 2) (\lambda V2t \in$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (12)$$

**Definition 12** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2Epair\_2E\_2C (ty\_2Epair\_2Eprod A\_27a A\_27b) x y))$

Let  $c\_2Efrac\_2Eabs\_frac : \iota$  be given. Assume the following.

$$c\_2Efrac\_2Eabs\_frac \in (ty\_2Efrac\_2Efrac)^{(ty\_2Epair\_2Eprod ty\_2Einteger\_2Eint ty\_2Einteger\_2Eint)} \quad (13)$$

**Definition 13** We define  $c\_2Efrac\_2Efrac\_ainv$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.(ap\ c\_2Efrac\_2Eabs\_fr)$

Let  $c\_2Einteger\_2Etint\_mul : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)} \quad (14)$$

**Definition 14** We define  $c\_2Einteger\_2Eint\_mul$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

Let  $c\_2Einteger\_2Etint\_add : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)} \quad (15)$$

**Definition 15** We define  $c\_2Einteger\_2Eint\_add$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 16** We define  $c\_2Efrac\_2Efrac\_add$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

**Definition 17** We define  $c\_2Efrac\_2Efrac\_sub$  to be  $\lambda V0f1 \in ty\_2Efrac\_2Efrac.\lambda V1f2 \in ty\_2Efrac\_2Efrac$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (16)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (17)$$

**Definition 18** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Einteger\_2Etint\_lt : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Etint\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)} \quad (18)$$

**Definition 19** We define  $c\_2Einteger\_2Eint\_lt$  to be  $\lambda V0T1 \in ty\_2Einteger\_2Eint.\lambda V1T2 \in ty\_2Einteger$

**Definition 20** We define  $c\_2Einteger\_2Eint\_sub$  to be  $\lambda V0x \in ty\_2Einteger\_2Eint.\lambda V1y \in ty\_2Einteger\_2Eint$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (19)$$

Let  $c\_2Einteger\_2Eint\_of\_num : \iota$  be given. Assume the following.

$$c\_2Einteger\_2Eint\_of\_num \in (ty\_2Einteger\_2Eint^{ty\_2Enum\_2Enum}) \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \text{True})) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Einteger\_2Eint. (\forall V1b \in ty\_2Einteger\_2Eint. \\ & ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\ c_2Enum_2E0)) V1b)) \Rightarrow ((ap c_2Efrac_2Efrac_nmr (ap c_2Efrac_2Eabs_frac \\ (ap (ap (c_2Epair_2E_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) \\ V0a) V1b)) = V0a)))))) \\ & \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Einteger\_2Eint. (\forall V1b \in ty\_2Einteger\_2Eint. \\ & ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\ c_2Enum_2E0)) V1b)) \Rightarrow ((ap c_2Efrac_2Efrac_dnm (ap c_2Efrac_2Eabs_frac \\ (ap (ap (c_2Epair_2E_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) \\ V0a) V1b)) = V1b)))))) \\ & \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Einteger\_2Eint. (\forall V1y \in ty\_2Einteger\_2Eint. \\ & ((ap (ap c_2Einteger_2Eint_sub V0x) V1y) = (ap (ap c_2Einteger_2Eint_add \\ V0x) (ap c_2Einteger_2Eint_neg V1y)))))) \\ & \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & ((\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\ & ((ap (ap c_2Einteger_2Eint_mul (ap c_2Einteger_2Eint_of_num \\ V0m)) (ap c_2Einteger_2Eint_of_num V1n)) = (ap c_2Einteger_2Eint_of_num \\ (ap (ap c_2Earithmetic_2E_2A V0m) V1n)))))) \wedge ((\forall V2x \in ty\_2Einteger\_2Eint. \\ & (\forall V3y \in ty\_2Einteger\_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\ (ap c_2Einteger_2Eint_neg V2x)) V3y) = (ap c_2Einteger_2Eint_neg \\ (ap (ap c_2Einteger_2Eint_mul V2x) V3y)))))) \wedge ((\forall V4x \in ty\_2Einteger\_2Eint. \\ & (\forall V5y \in ty\_2Einteger\_2Eint. ((ap (ap c_2Einteger_2Eint_mul \\ V4x) (ap c_2Einteger_2Eint_neg V5y)) = (ap c_2Einteger_2Eint_neg \\ (ap (ap c_2Einteger_2Eint_mul V4x) V5y)))))) \wedge ((\forall V6x \in ty\_2Einteger\_2Eint. \\ & ((ap c_2Einteger_2Eint_neg (ap c_2Einteger_2Eint_neg V6x)) = \\ & V6x))))))) \\ & \end{aligned} \quad (26)$$

### Theorem 1

$$\begin{aligned}
 & (\forall V0a1 \in ty\_2Einteger\_2Eint. (\forall V1b1 \in ty\_2Einteger\_2Eint. \\
 & (\forall V2a2 \in ty\_2Einteger\_2Eint. (\forall V3b2 \in ty\_2Einteger\_2Eint. \\
 & ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
 & c_2Enum_2E0)) V1b1)) \Rightarrow ((p (ap (ap c_2Einteger_2Eint_lt (ap c_2Einteger_2Eint_of_num \\
 & c_2Enum_2E0)) V3b2)) \Rightarrow ((ap (ap c_2Efrac_2Efrac_sub (ap c_2Efrac_2Eabs_frac \\
 & (ap (ap (c_2Epair_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) \\
 & V0a1) V1b1))) (ap c_2Efrac_2Eabs_frac (ap (ap (c_2Epair_2C \\
 & ty_2Einteger_2Eint ty_2Einteger_2Eint) V2a2) V3b2))) = (ap c_2Efrac_2Eabs_frac \\
 & (ap (ap (c_2Epair_2C ty_2Einteger_2Eint ty_2Einteger_2Eint) \\
 & (ap (ap c_2Einteger_2Eint_sub (ap (ap c_2Einteger_2Eint_mul \\
 & V0a1) V3b2)) (ap (ap c_2Einteger_2Eint_mul V2a2) V1b1))) (ap ( \\
 & ap c_2Einteger_2Eint_mul V1b1) V3b2)))))))))))
 \end{aligned}$$